A Machine Learning enhanced Variable Neighborhood Search approach for the Uncapacitated Facility Location problem

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INTERNATIONAL CONFERENCE ON VARIABLE NEIGHBORHOOD SEARCH VNS 2025

MINISTERIO DE CIENCIA E INNOVACIÓN

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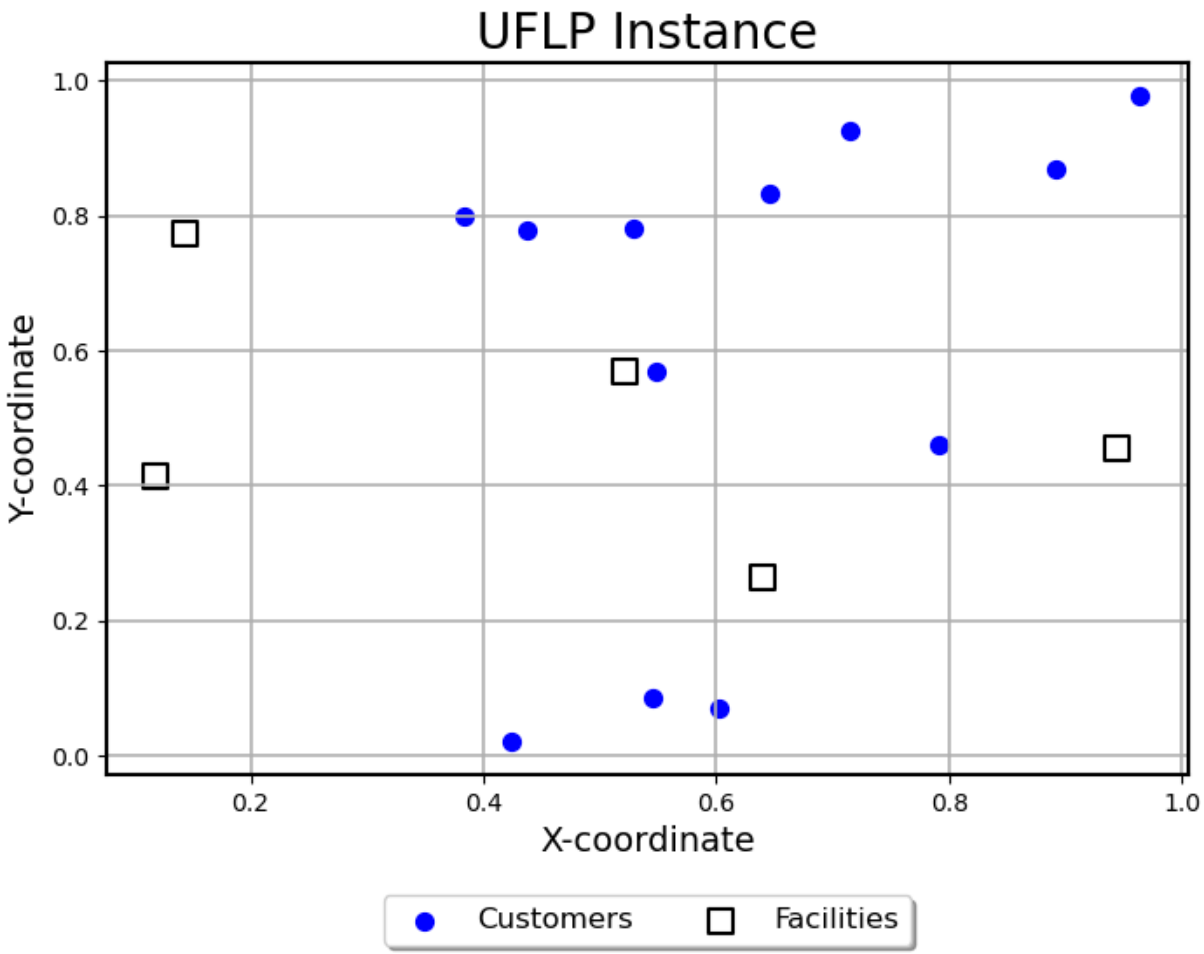
Outline

Introduction

- The **UFLP** (Uncapacitated Facility Location Problem) belongs to the family of **Facility Location Problems**.
- It aims to **minimize** the sum of the **cost of opening facilities** plus the **cost of assigning customers** to these facilities.
- Real-world applications:
  - Resource allocation and network configuration.
  - Logistics and transportation.

Introduction

Solution representation



- Set of **customers**
- Set of **facilities**

Introduction

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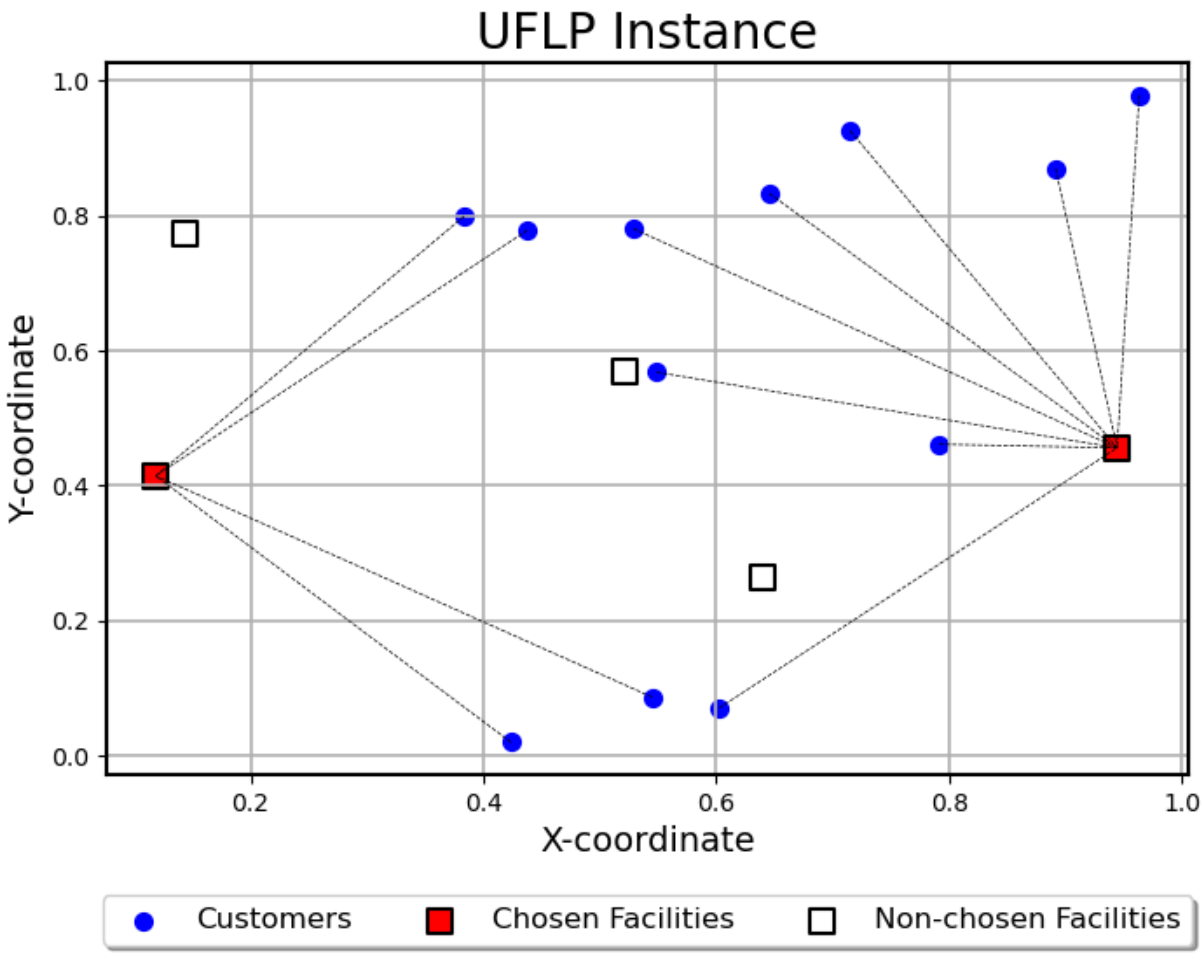

- Set of **customers**
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Solution: open an unbounded number of facilities and assign customers.

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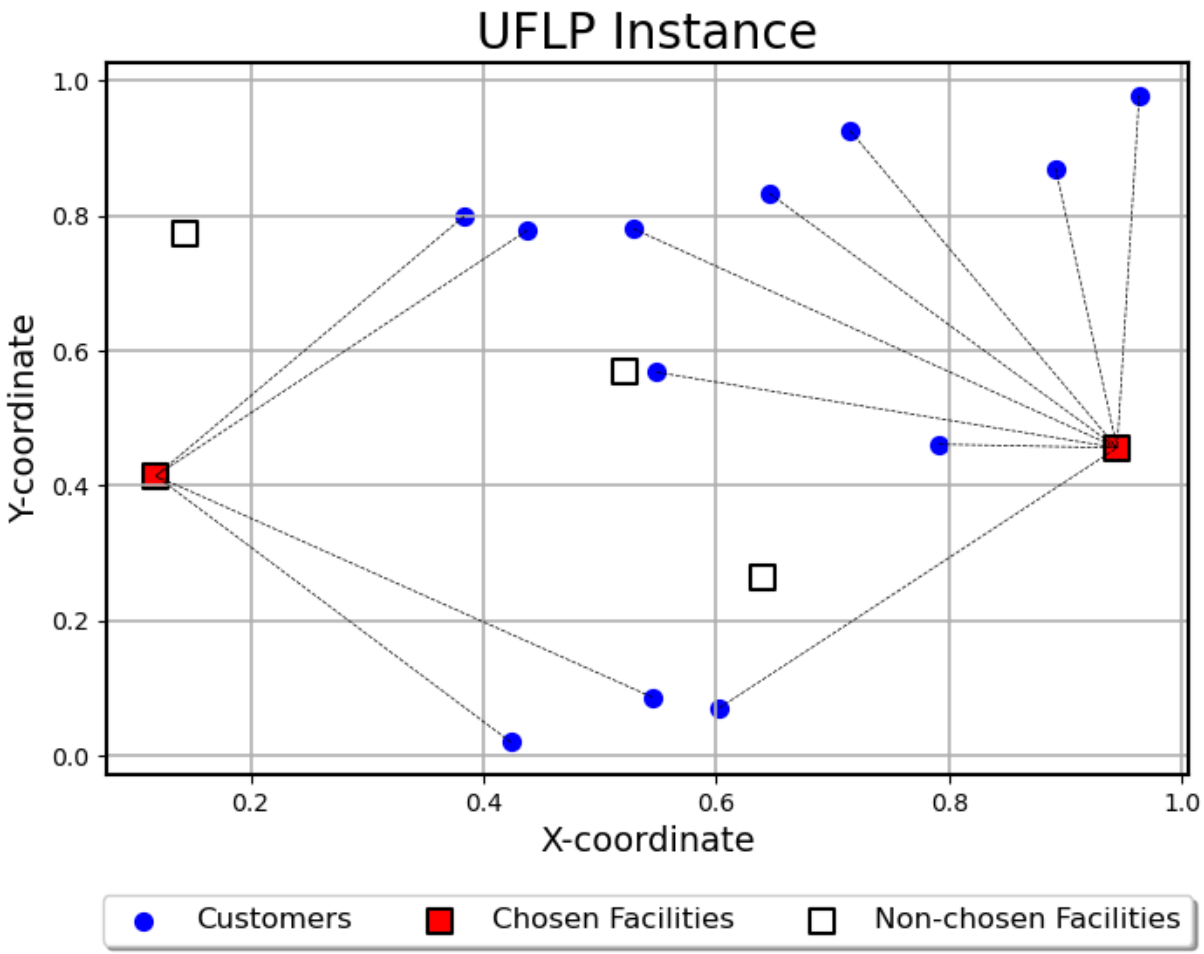


- Set of **customers**
- Set of **facilities**

Solution: open an unbounded number of facilities and assign customers.

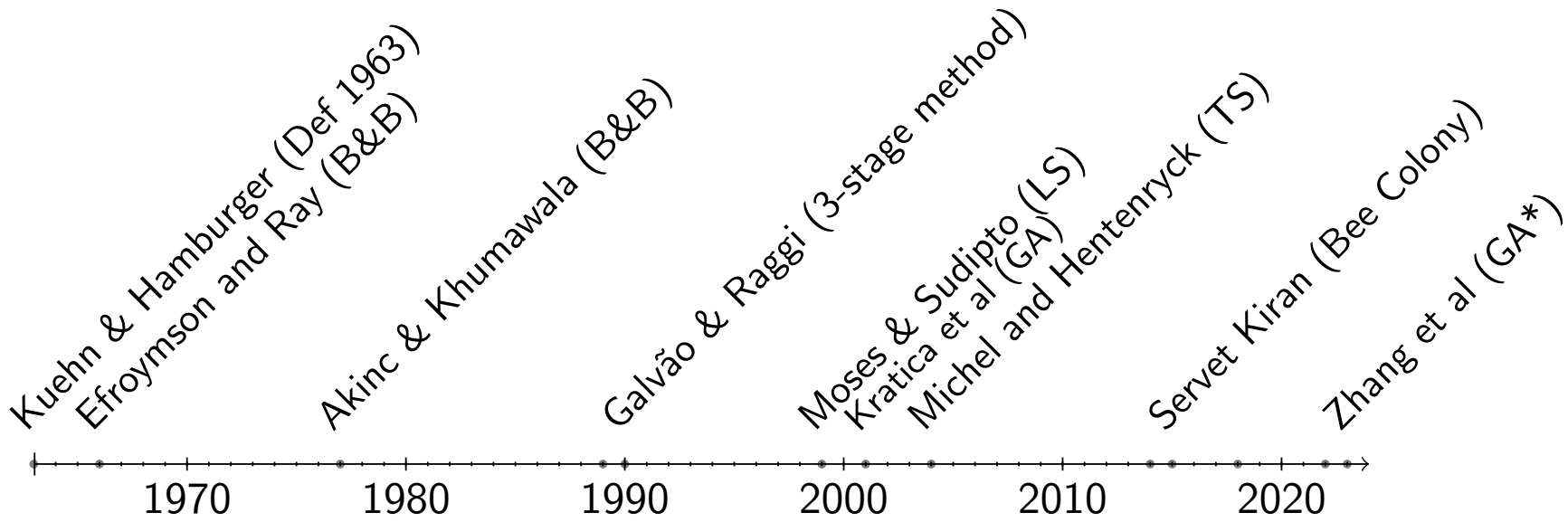
Objective: minimize opening cost plus service cost.

$$\sum_{i=1}^{m}\sum_{j=1}^{n} d_{ij}y_{ij} + \sum_{j=1}^{n} g_j x_j$$

Introduction

Literature review

It has been proven to be $\mathcal{NP}$-Hard (Karp 1972).



Introduction

Mathematical formulation

$$\text{Minimize} \quad \sum_{i=1}^{m}\sum_{j=1}^{n} d_{ij}y_{ij} + \sum_{j=1}^{n} g_j x_j \tag{1}$$

Subject to

$$\sum_{j=1}^{n} y_{ij} = 1, \quad i = 1, 2, \ldots, m \tag{2}$$

$$y_{ij} \leq x_j, \quad i = 1, 2, \ldots, m, \quad j = 1, 2, \ldots, n \tag{3}$$

$$y_{ij}, x_j \in \{0, 1\}, \quad i = 1, 2, \ldots, m, \quad j = 1, 2, \ldots, n \tag{4}$$

Implemented in Gurobi.

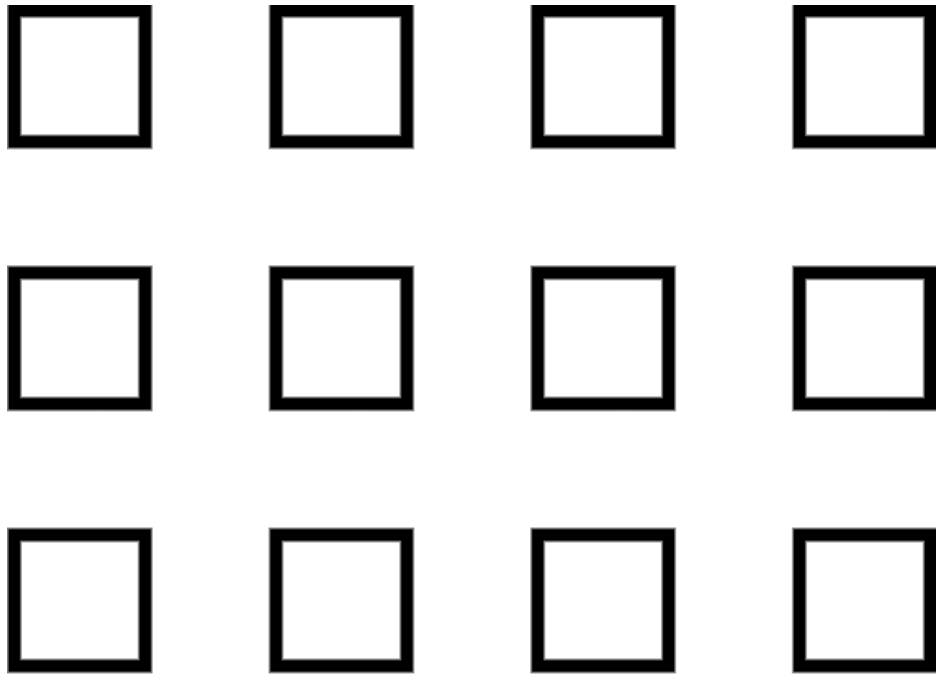
BVNS

Algorithm 1 BVNS(k_{max})

```
1: S ← Constructive()
2: S* ← S
3: k ← 0
4: while k < k_max do
5:     S ← Shake(S,k)
6:     S ← Improvement(S)
7:     if F(S) < F(S*) then
8:         S* ← S
9:         k ← 1
10:    else
11:        k ← k + 1
12:    end if
13: end while
14: return S*
```

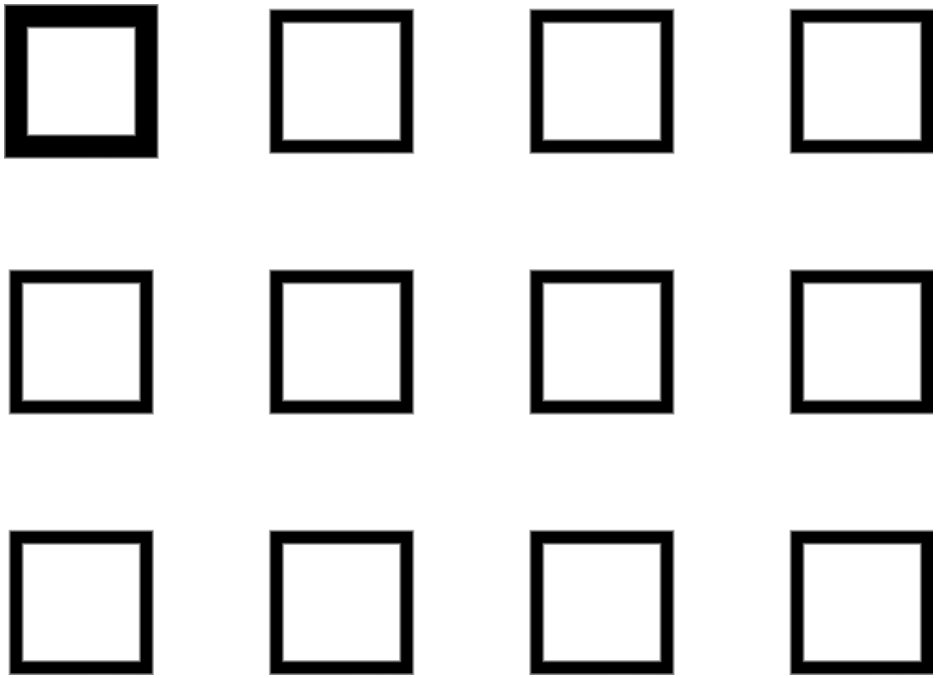
Constructive

Random constructive



Constructive

Random constructive



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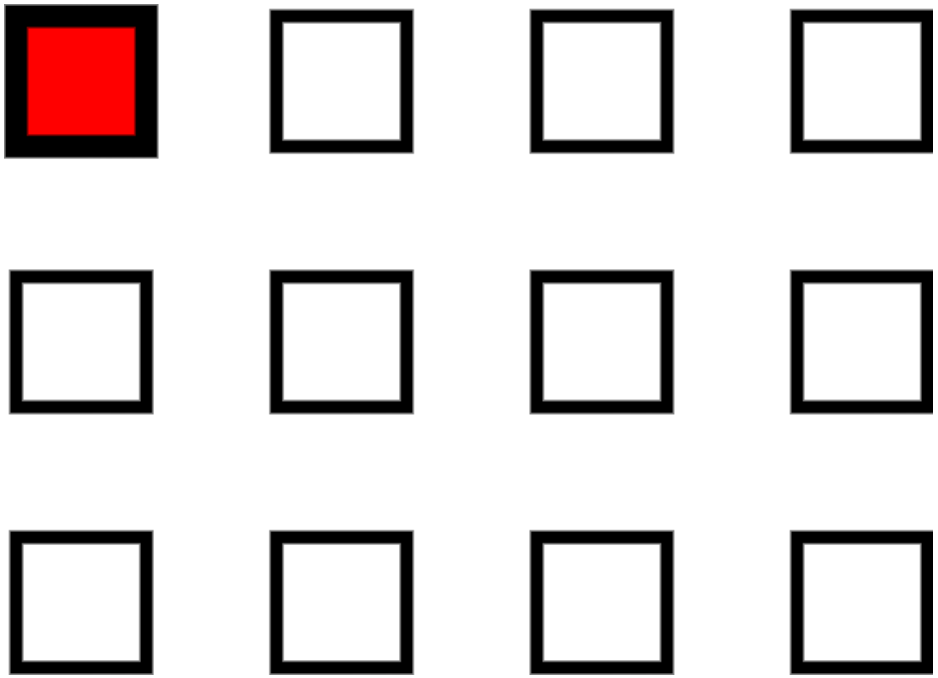

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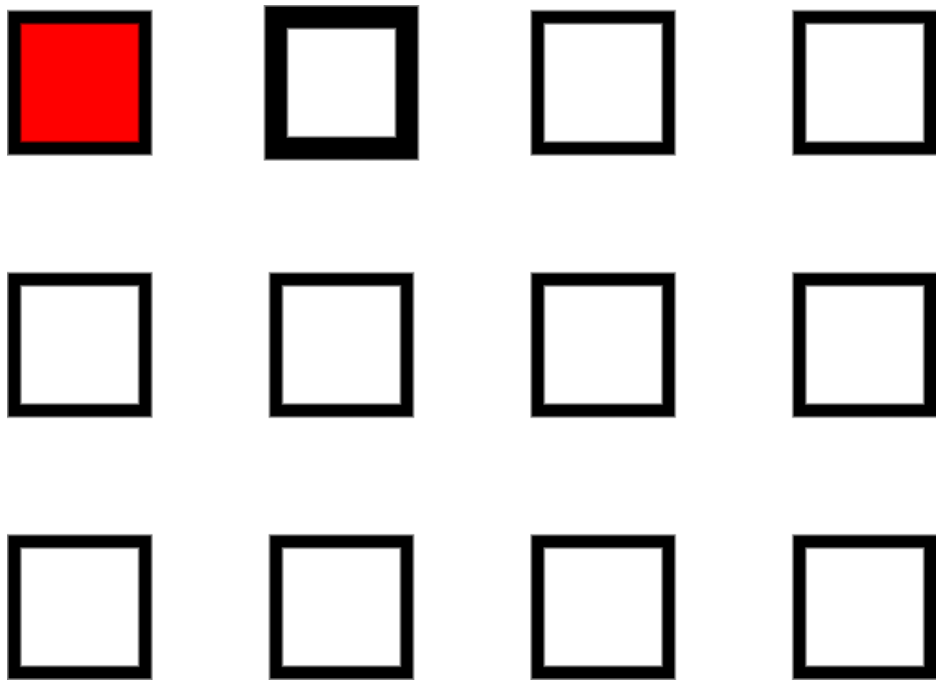

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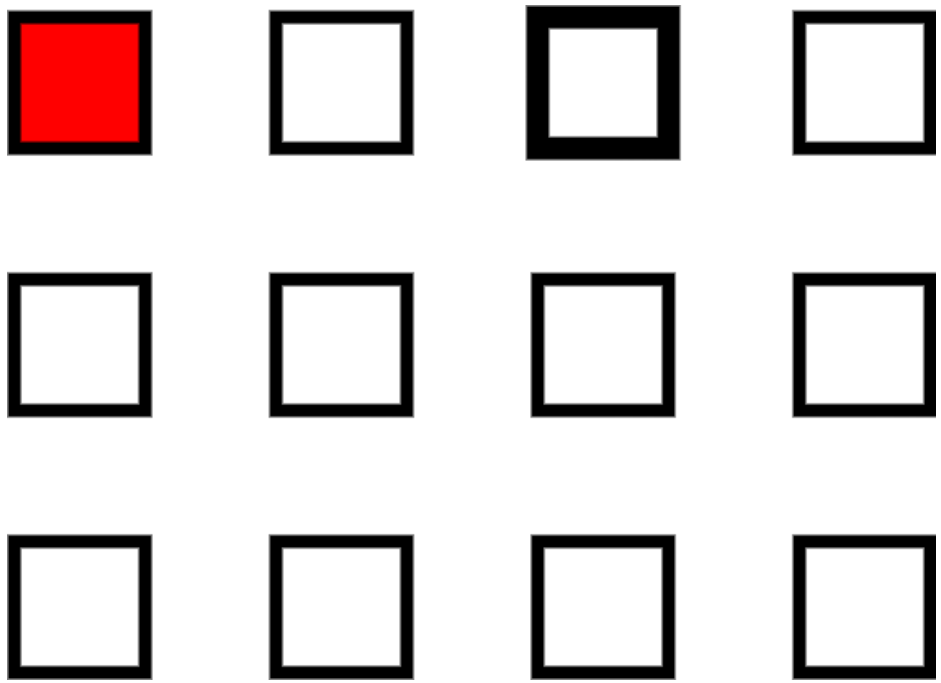

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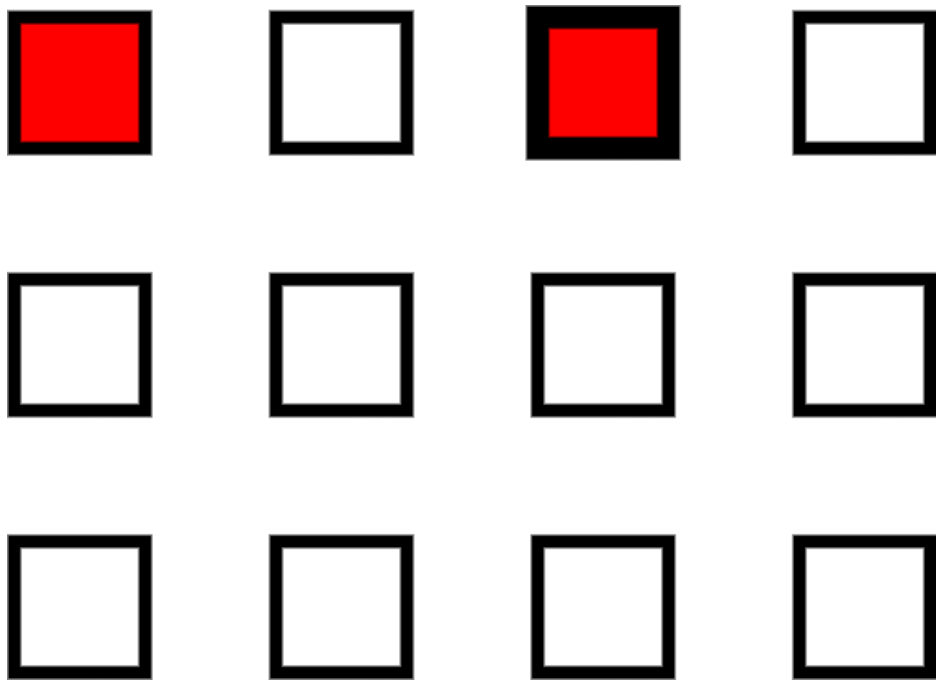

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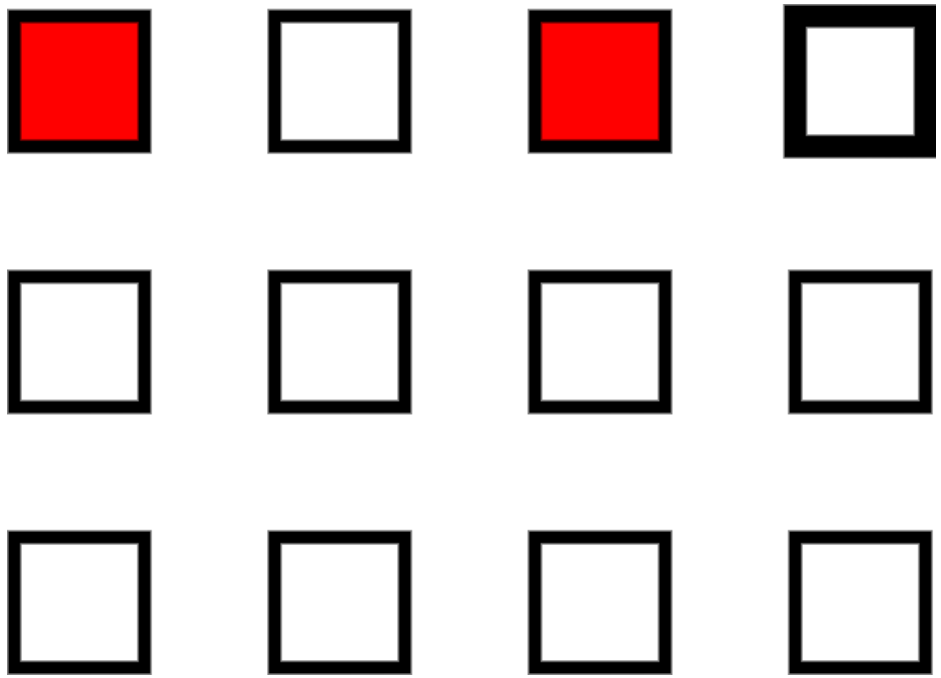

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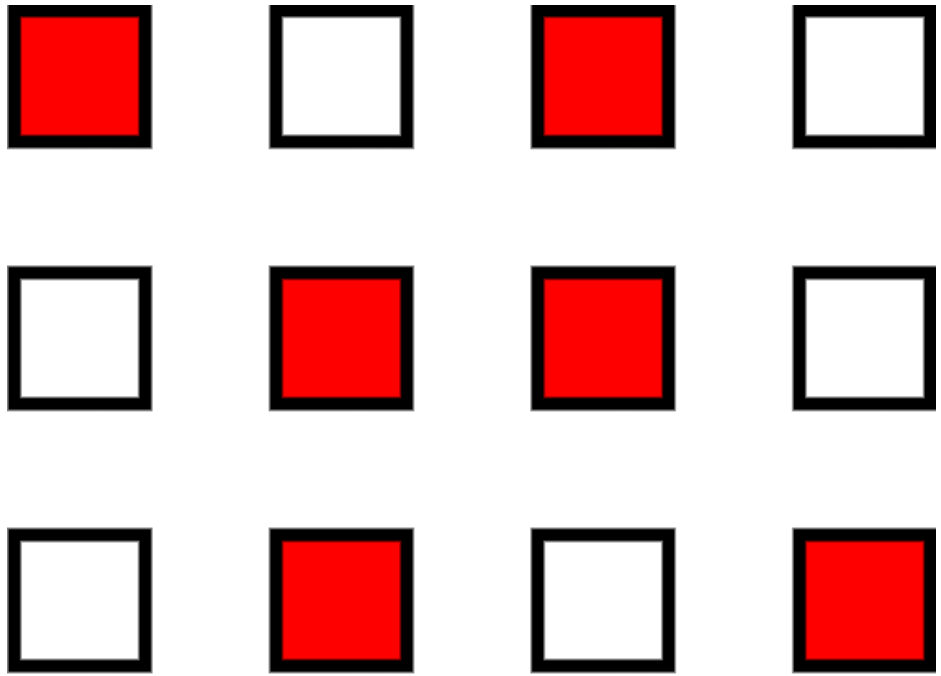
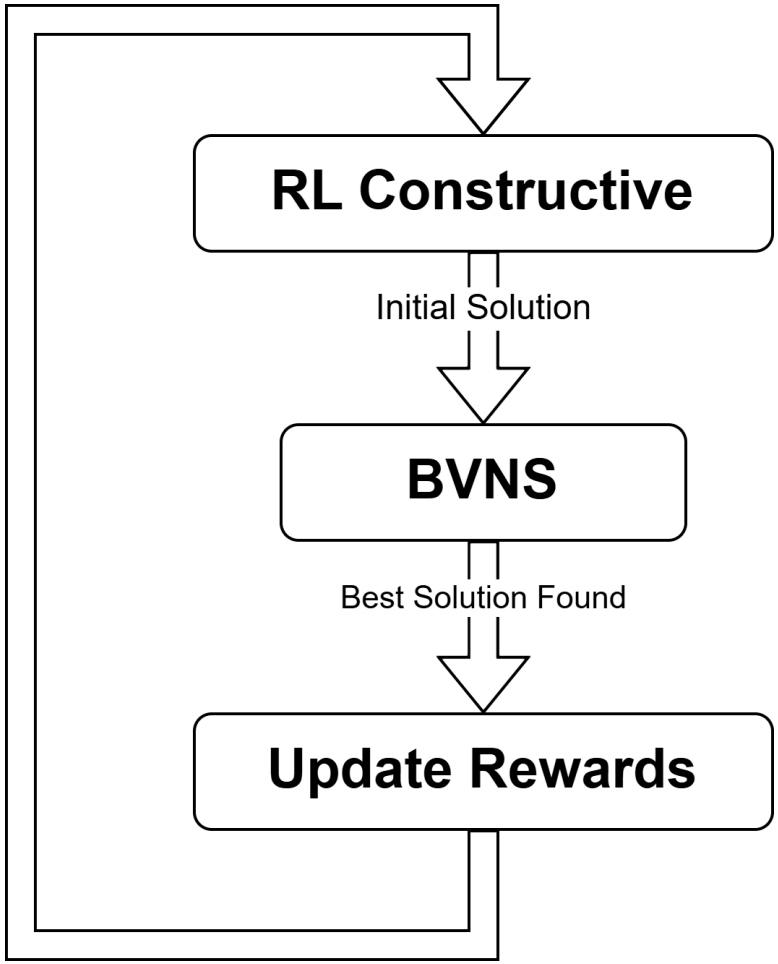


Reinforcement Learning Constructive



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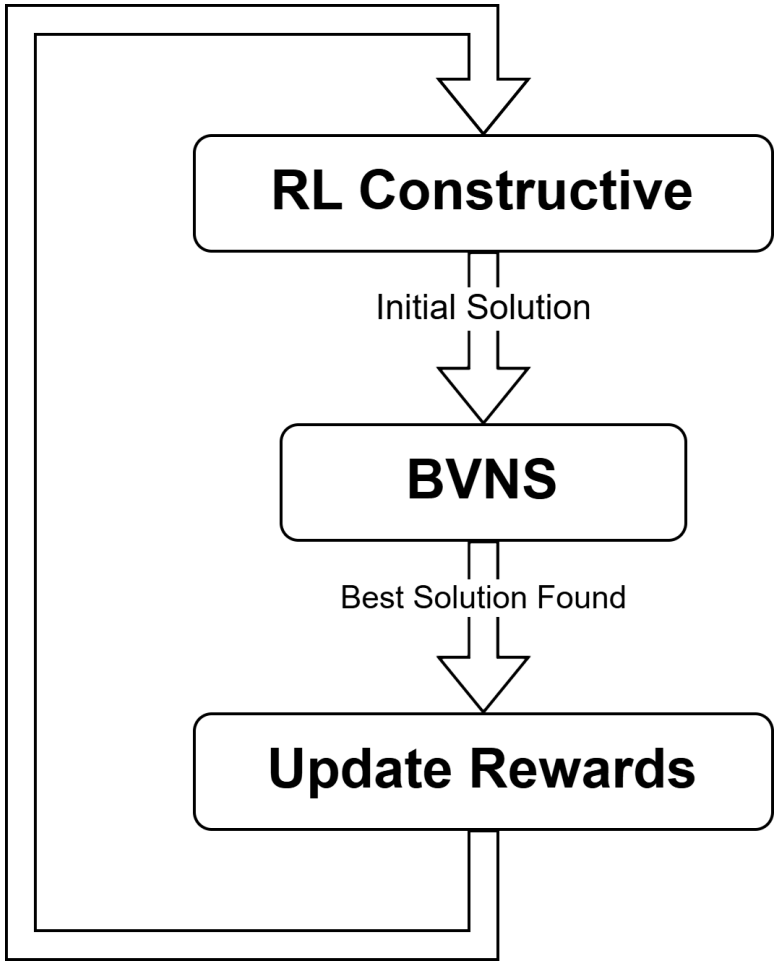


Reward Update

$$R_{k+1}(i) = R_k(i) + \Delta R(i)$$

where for $i = 1, \ldots, N$:

$$\Delta R(i) = \begin{cases} +1/N & \text{if } i \in S^* \\ -1/N & \text{if } i \notin S^* \end{cases}$$

Reinforcement Learning Constructive

Algorithm 2 `RLConstructive(N)`

1: $S' \leftarrow \emptyset$
2: **for** $i = 1$ **to** N **do**
3: **if** `nextRandom(0,1)` $<$ `Reward(i)` **then**
4: $S \leftarrow S \cup \{i\}$
5: **end if**
6: **end for**
7: **return** S'

Improvement method

Algorithm 3 `Improvement(S)`

1: $improvement \leftarrow$ `True`
2: **while** *improvement* **do**
3: $improvement \leftarrow$ `False`
4: $x \leftarrow \arg\min_{i \in S} \mathcal{F}(S \setminus \{i\})$
5: $S' \leftarrow S \setminus \{x\}$
6: **if** $\mathcal{F}(S') < \mathcal{F}(S)$ **then**
7: $improvement \leftarrow$ `True`
8: $S \leftarrow S'$
9: **end if**
10: **end while**
11: **return** S

Shake

Algorithm 4 `Shake(`S`,`k`)`

1: $C \leftarrow \{ a_i : a_i \in A \wedge a_i \notin S\}$
2: $C' \leftarrow$ `RandomSelection(C,k)`
3: $S \leftarrow S \cup C'$
4: **return** S

Results

- Programming languages: **Java 21**, **Python 3.8** and **Gurobi 11**.
- Server characteristics: AMD EPYC 7643 32-core with 32 GB of RAM.
- Instances: 15 CAP (66-1100 nodes), 87 K90 (502-914 nodes) and 700 G700 (3006-3906 nodes).
- Metrics:
  - $\mathcal{F}$: objective function value.
  - **Time (s)**: execution time in seconds.
  - **GAP (%)**: deviation from the exact value.
  - **# Opt**: total number of instances where the algorithm achieves the optimal solution.

Preliminary results

Random constructive

Local Search	K	$\mathcal{F}$	Time (s)	GAP (%)	# Opt
First Improvement	0.1	3 174 785.33	792.34	0.274205	58
	0.2	3 170 779.44	1007.23	0.233152	66
	0.3	3 171 017.58	1240.25	0.243865	65
	0.4	3 171 210.25	1502.95	0.244462	66
	0.5	3 170 757.61	1728.05	0.233776	65
Best Improvement	0.1	3 167 364.00	**93.92**	0.006122	133
	0.2	3 167 281.28	227.00	0.001724	148
	0.3	3 167 279.87	423.14	0.001707	**151**
	0.4	3 167 278.90	684.37	0.001595	**151**
	0.5	**3 167 276.27**	1017.10	**0.001489**	150

Table 1: Results of the BVNS algorithm on 20% of the dataset.

Preliminary results

Reinforcement learning constructive

Local Search	K	$\mathcal{F}$	Time (s)	GAP (%)	# Opt
First Improvement	0.1	3 177 592.01	939.49	0.497938	58
	0.2	3 172 158.29	1093.90	0.245668	65
	0.3	3 171 408.92	1316.49	0.239123	66
	0.4	3 174 379.58	1598.42	0.262672	63
	0.5	3 170 757.10	1853.15	0.232076	63
Best Improvement	0.1	3 169 410.98	**88.01**	0.210642	139
	0.2	3 167 271.68	224.93	0.001082	**151**
	0.3	**3 167 269.01**	424.91	**0.000836**	**151**
	0.4	3 167 276.50	683.34	0.001309	147
	0.5	3 167 271.70	1020.71	0.001015	149

Table 2: Results of the BVNS-RL algorithm on 20% of the dataset.

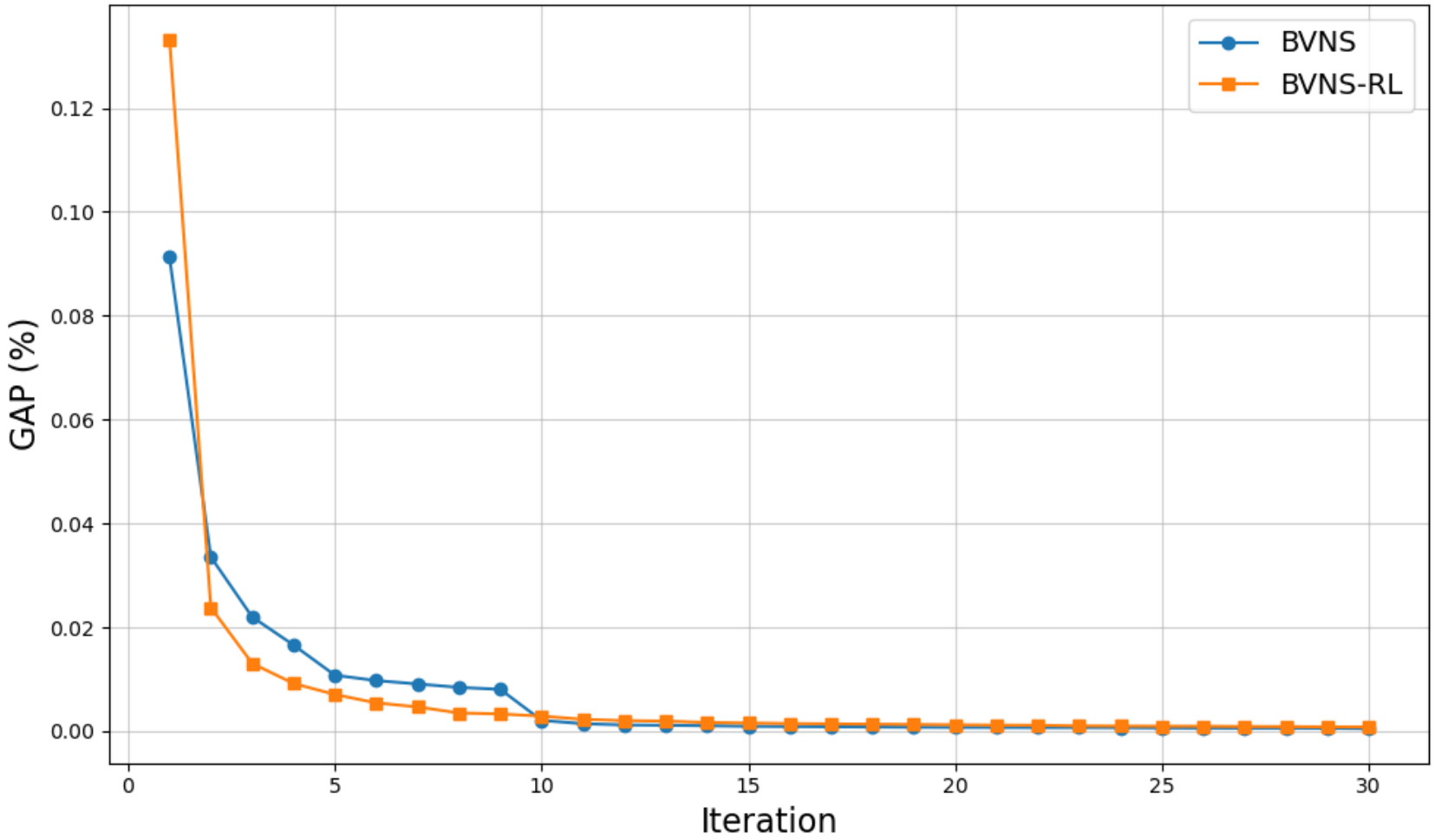
Final results

Instances	Algorithm	$\mathcal{F}$	Time (s)	GAP (%)	# Opt
Cap	EGTOA (Zhang)	3 502 545.59	258.931	0.00000	15
	BVNS	3 502 796.74	0.254	0.00218	14
	BVNS-RL	3 502 545.59	0.208	0.00000	15
	Exact Model	3 502 545.59	0.802	0.00000	15
K90	EGTOA (Zhang)	3 069 045.61	1 405.238	12.12858	0
	BVNS	2 765 428.69	3.825	0.00568	71
	BVNS-RL	2 765 390.11	3.508	0.00374	73
	Exact Model	2 765 298.57	2.642	0.00000	87
G700	EGTOA (Zhang)	27 202 096.73	3 597.980	174.81629	23
	BVNS	2 943 809.46	248.588	0.00018	666
	BVNS-RL	2 943 809.24	239.329	0.00010	678
	Exact Model	2 943 809.00	62.861	0.00000	700
Aggregated	EGTOA (Zhang)	24 140 913.79	1 236.663	153.89849	38
	BVNS	2 934 913.80	217.392	0.00081	751
	BVNS-RL	2 934 904.73	209.275	0.00050	766
	Exact Model	**2 934 894.58**	**55.168**	**0.000000**	**802**

Table 3: Final results of the EGTOA, BVNS, and Exact algorithms.

Results

GAP Convergence



Conclusions

Conclusions:

- Two constructive heuristics were explored with the algorithm.
- **BVNS** consistently achieves competitive solutions, comparable to state-of-the-art.
- **Reinforcement Learning (RL)** showed an interesting fast progression towards better solutions, improving the performance of the random constructive.

Future work:

- Speed up the objective function calculation
- Test different machine learning methods to guide the solution search.

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