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Variable Neighborhood Search for the Minimum k -Dominating Set Problem

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Introduction

- The Minimum k -Dominating Set (MKDS) is an **NP-Hard** problem
- A **variant** from the classical domination problems
- **Real-world applications:**
 - Facility location problems
 - Social networks
 - Network designs

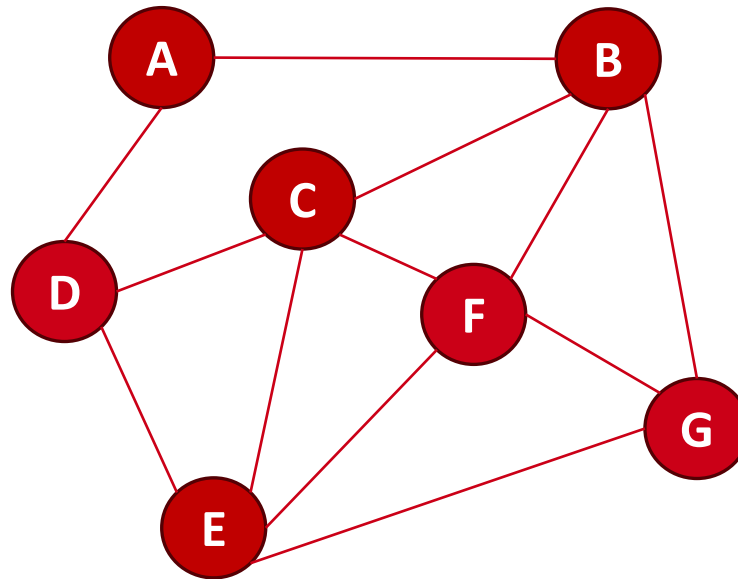
Introduction – Problem definition

- Given a graph $G = (V, E)$, consist in finding the **subset** $D_k \subseteq V$ **with minimum cardinality** such $\forall v \in V$ verify at **least one** of the following:
 - $v \in D_k$
 - v is adjacent to at least k vertices from D_k

Introduction – Exact model

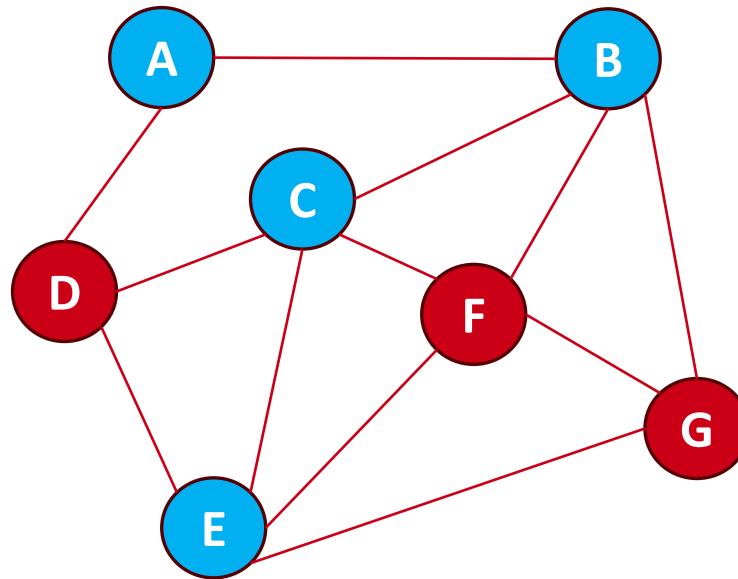
$$\begin{array}{ll} \min & \sum_{i \in V} x_i \\ \text{s. t.} & \sum_{j \in N(i)} x_j \geq k(1 - x_i) \quad \forall i \in V \\ & x_i \in \{0,1\} \quad \forall i \in V \end{array}$$

Introduction – Example



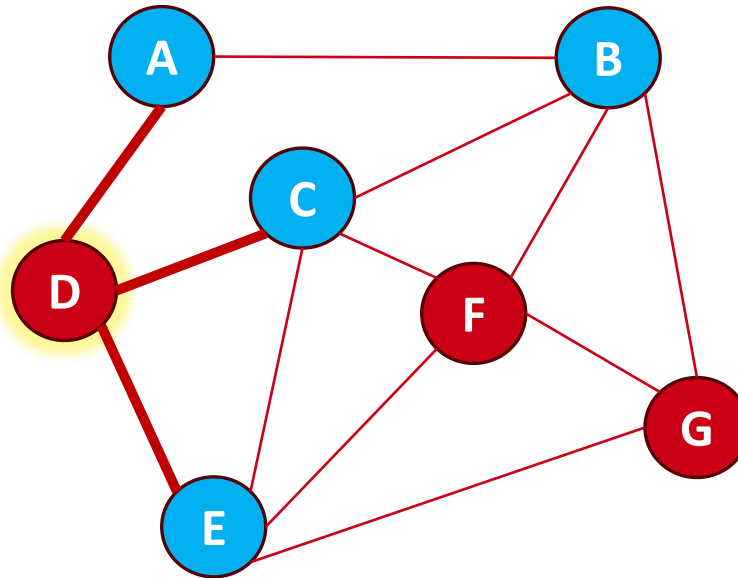
$$k = 2$$

Introduction – Example



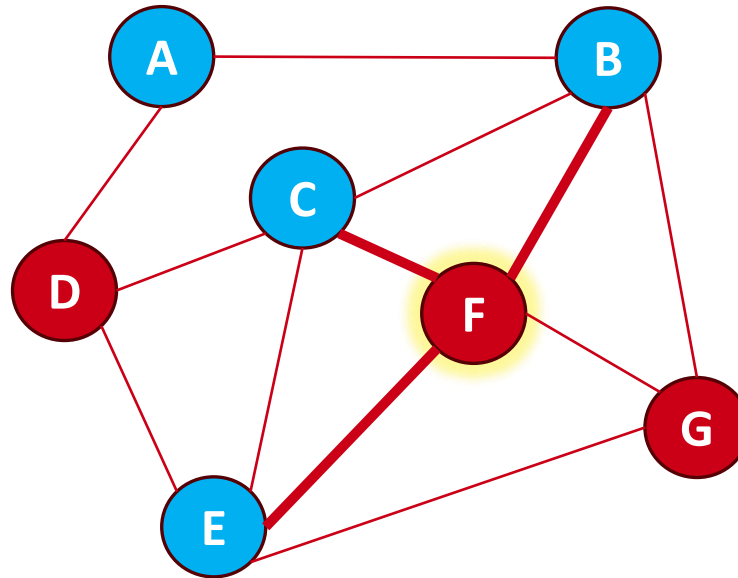
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Introduction – Example



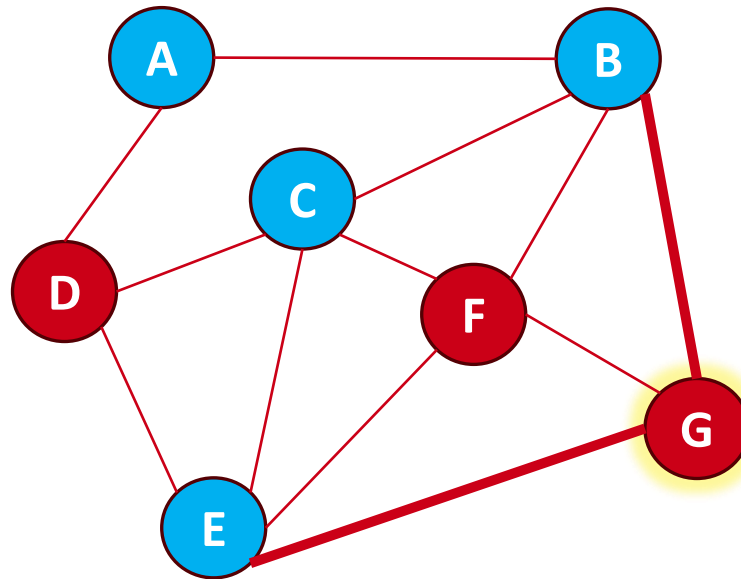
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Introduction – Example



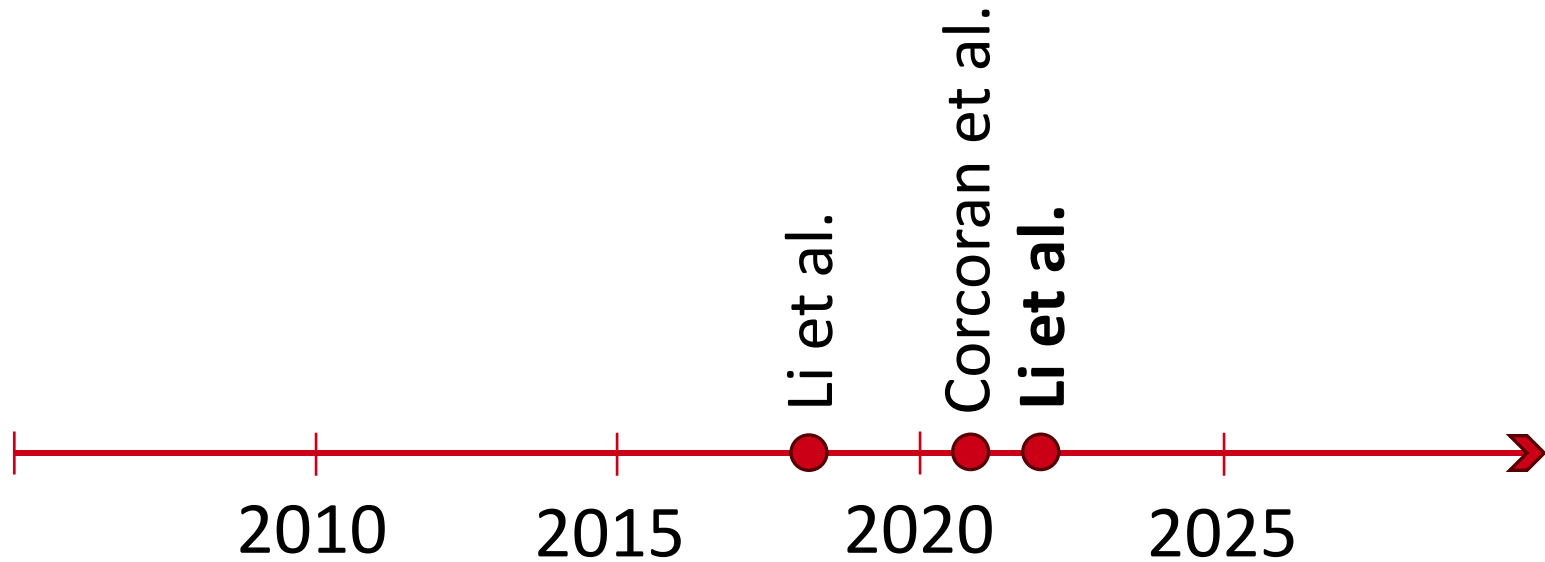
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Introduction – Example



$$k = 2$$

Introduction – Literature review



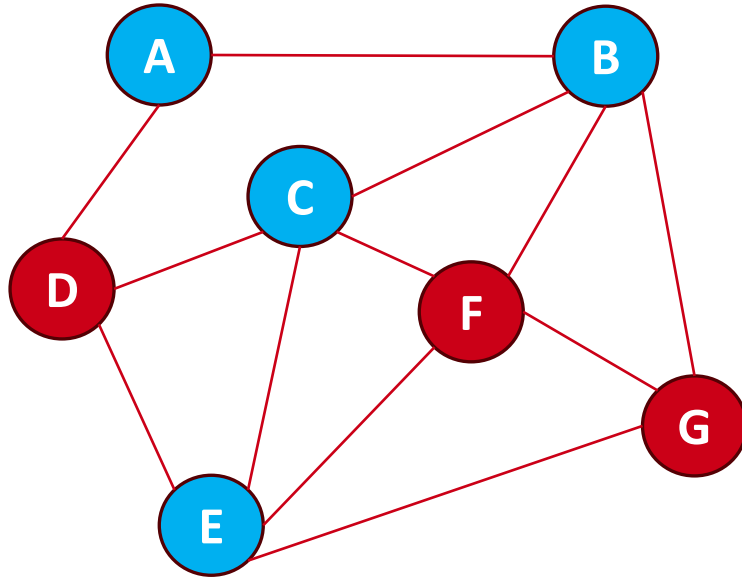
Algorithmic proposal – VNS

- Proposed by Nenad Mladenović y Pierre Hansen in 1997
- The **main components** of our proposal are:
 - **Initial solution**
 - **Postprocess**
 - Shake
 - **Improvement method**

Algorithmic proposal – Initial solution

- Initially it starts with $D_k = \{\}$
- We add the vertices with **less degree than k**
- Iteratively, until reaching a feasible solution, we add the vertex $v \notin D_k$ with **greatest coverage**
- **Coverage** of v : Number of vertices k -dominated after adding v to D_k

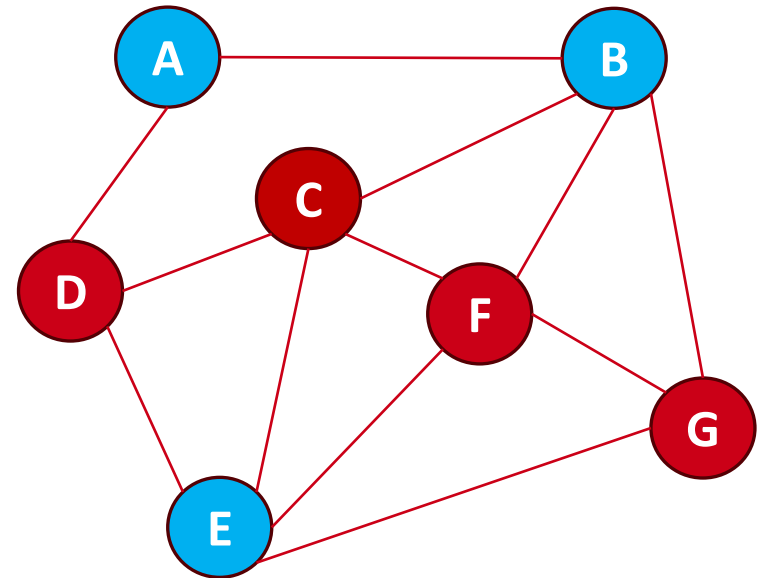
Algorithmic proposal – Postprocess



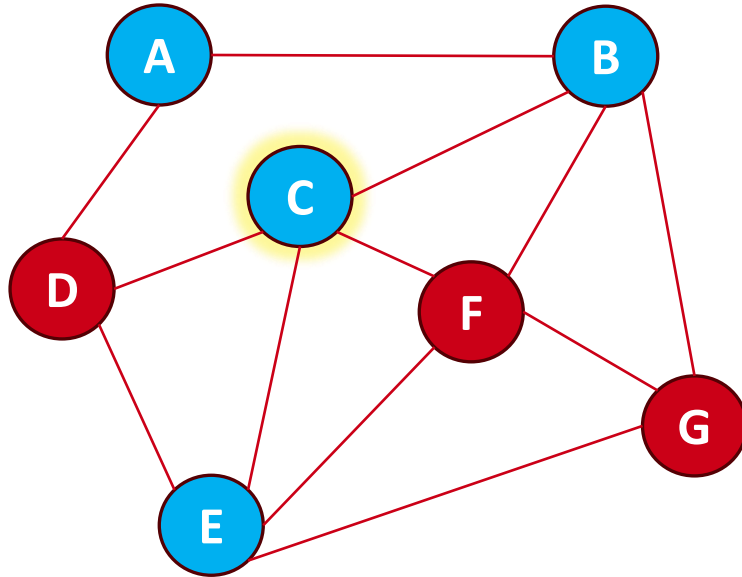
Vertex C is redundant

$k = 2$

Vertex C is removed



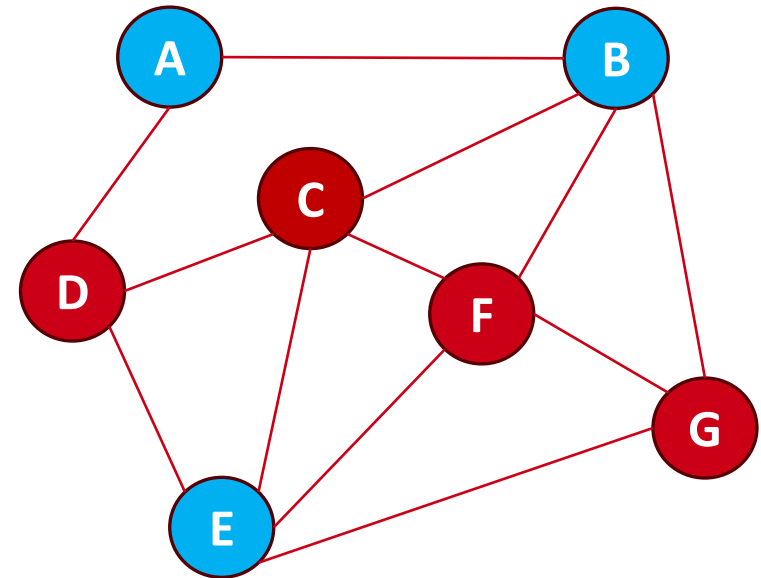
Algorithmic proposal – Postprocess



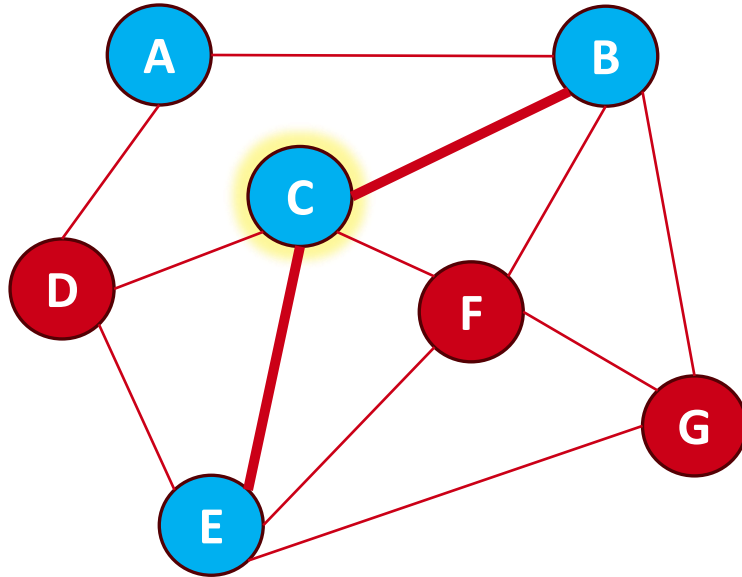
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Vertex C is removed



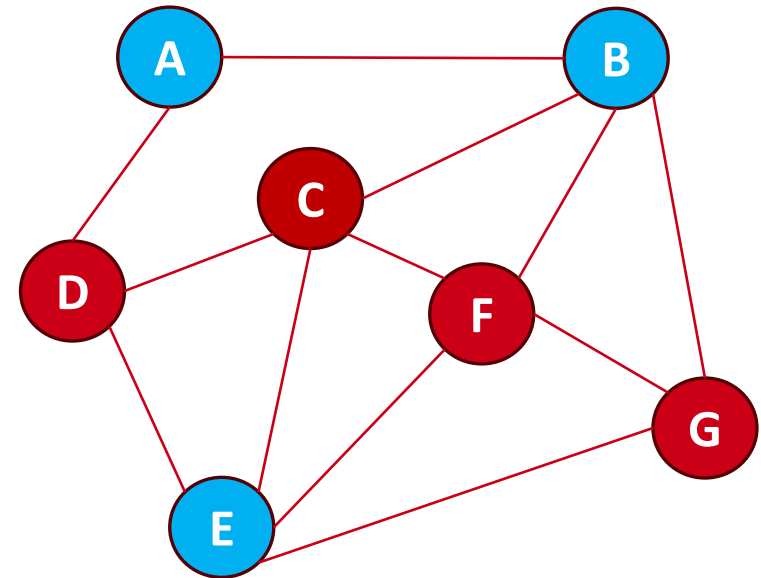
Algorithmic proposal – Postprocess



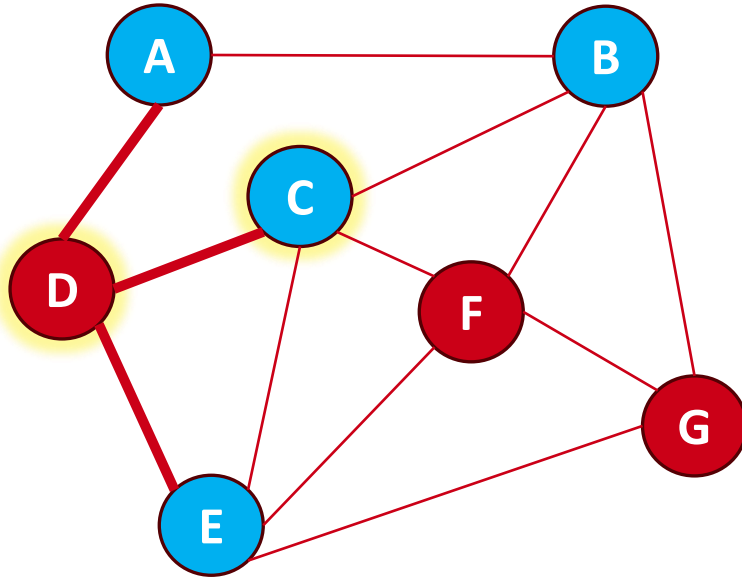
Vertex C is redundant

$k = 2$

Vertex C is removed



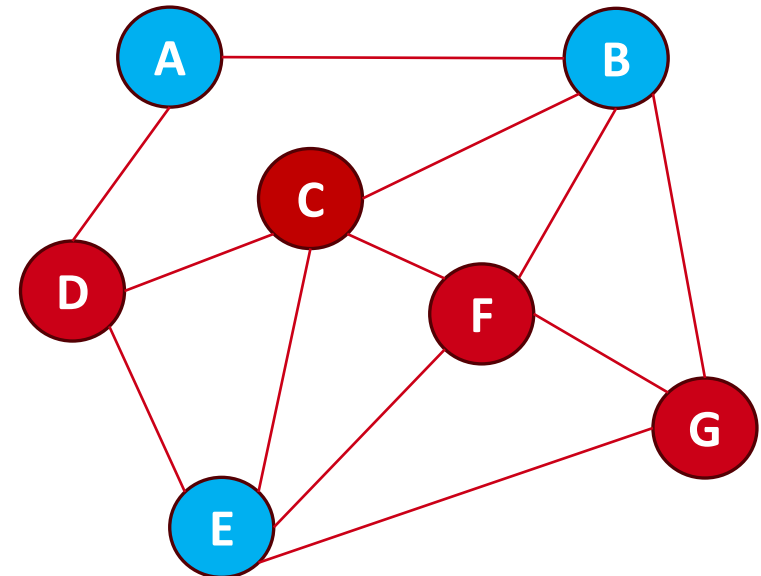
Algorithmic proposal – Postprocess



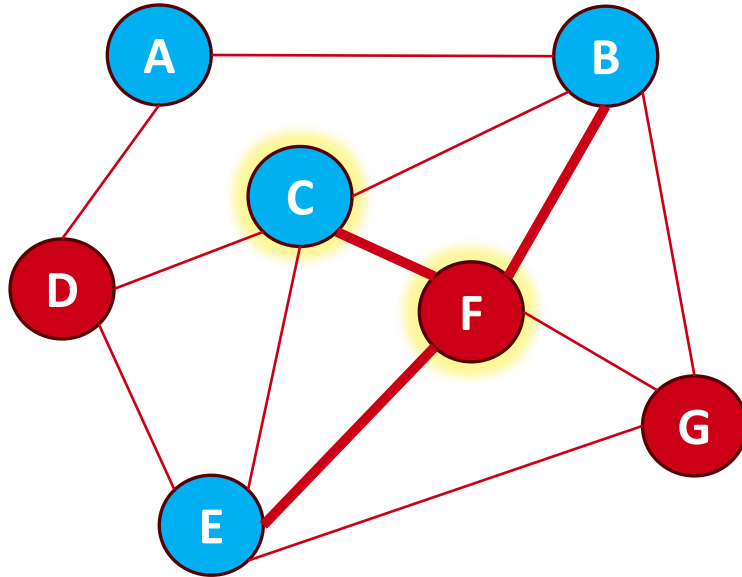
Vertex C is redundant

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Vertex C is removed



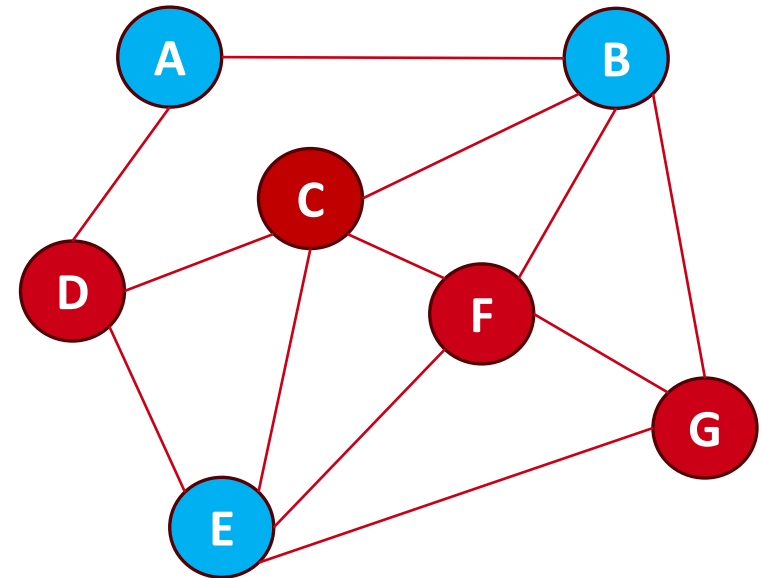
Algorithmic proposal – Postprocess



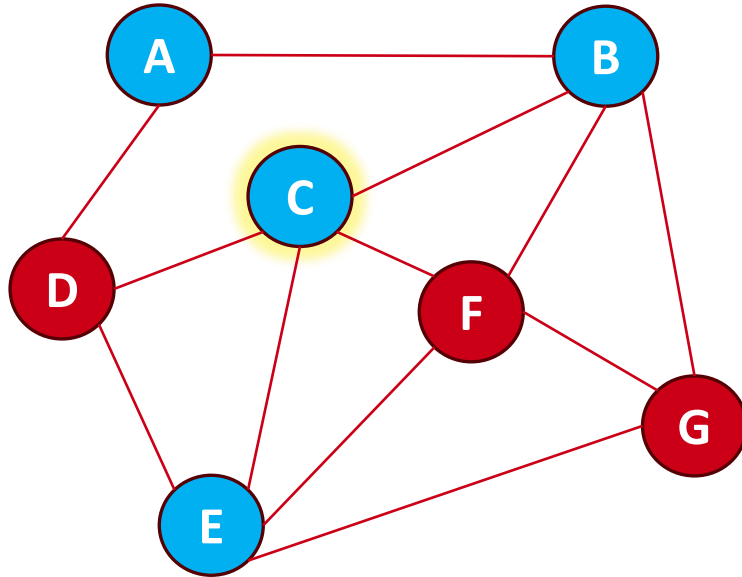
Vertex C is redundant

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Vertex C is removed



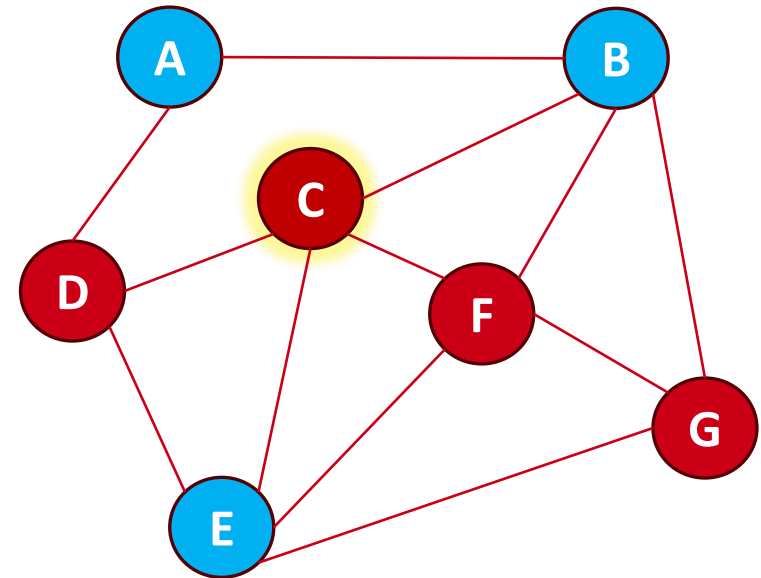
Algorithmic proposal – Postprocess



Vertex C is redundant

$k = 2$

Vertex C is removed



Algorithmic proposal – Improvement method

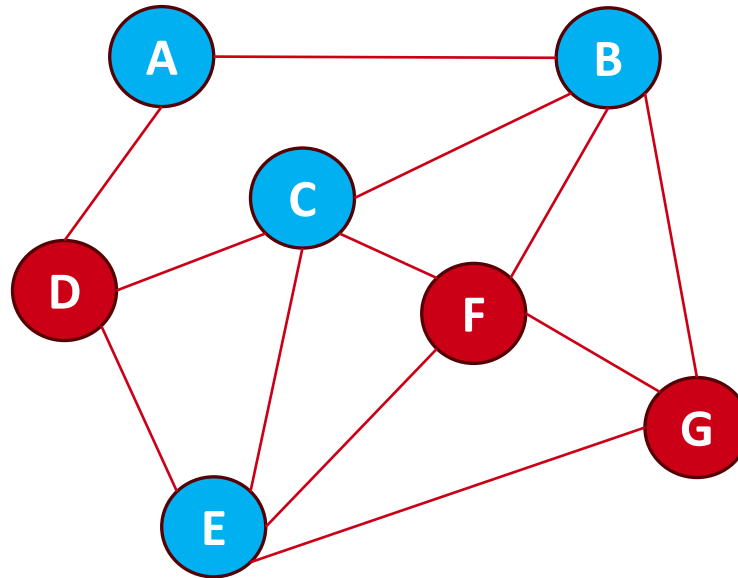
Two movements are proposed:

- **Exchange 1×1 :**
 - No improvement of the objective function
 - Many possible combinations
 - **Combined with the postprocess**

- **Exchange 2×1 :**
 - Improvement of the objective function
 - Less possible combinations

Algorithmic proposal – Improvement method

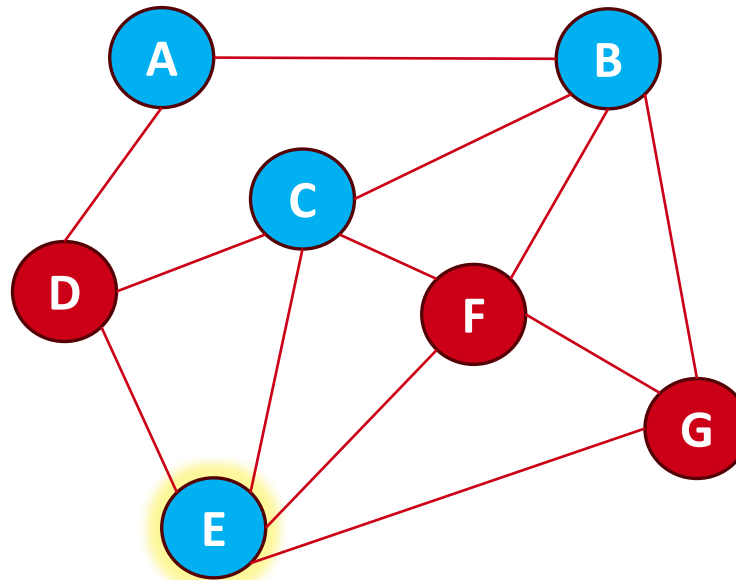
Exchange 1×1



$k = 2$

Algorithmic proposal – Improvement method

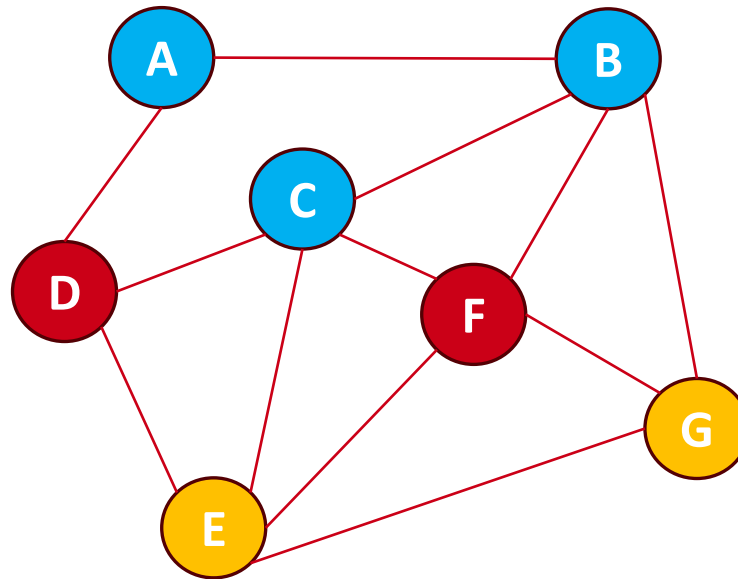
Exchange 1×1



$k = 2$

Algorithmic proposal – Improvement method

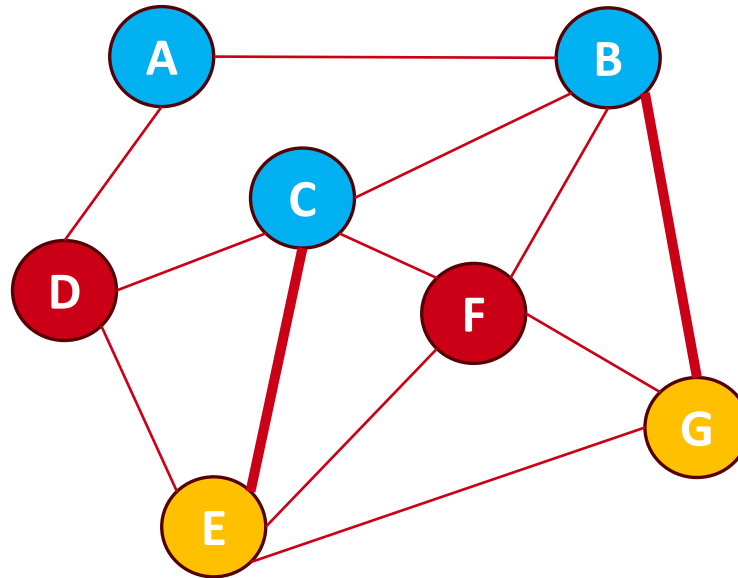
Exchange 1×1



$k = 2$

Algorithmic proposal – Improvement method

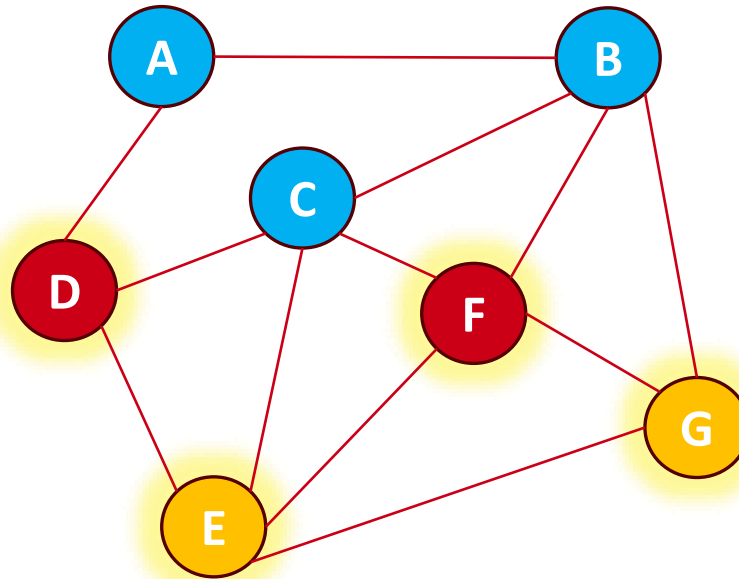
Exchange 1×1



$k = 2$

Algorithmic proposal – Improvement method

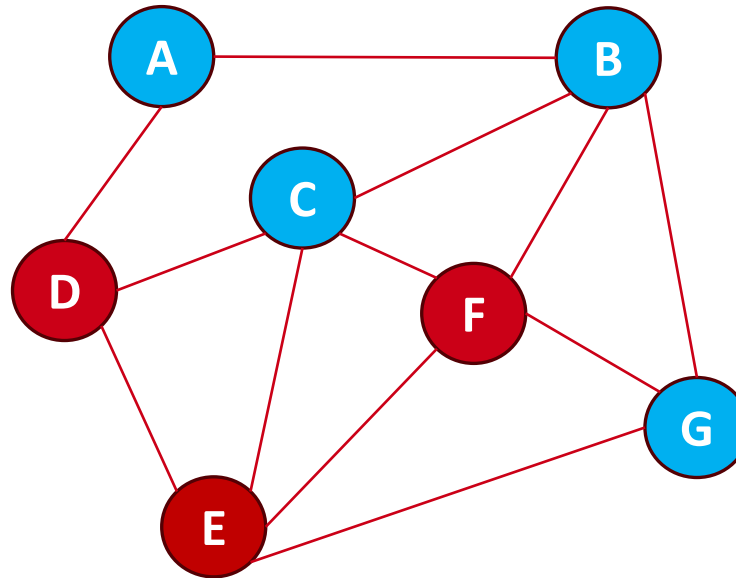
Exchange 1×1



$k = 2$

Algorithmic proposal – Improvement method

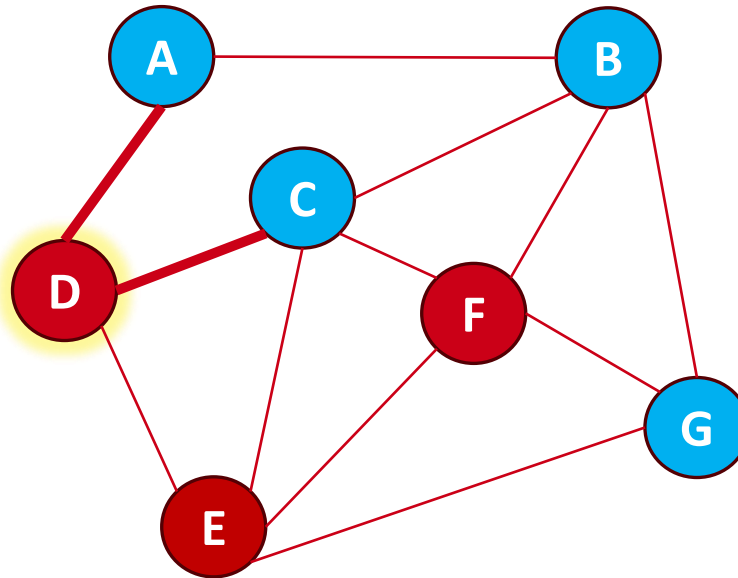
Exchange 1×1



$k = 2$

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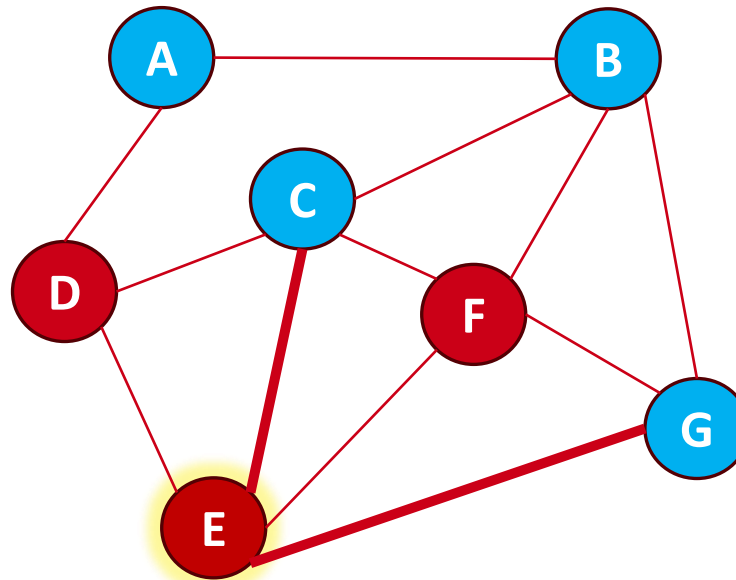
Exchange 1×1



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Algorithmic proposal – Improvement method

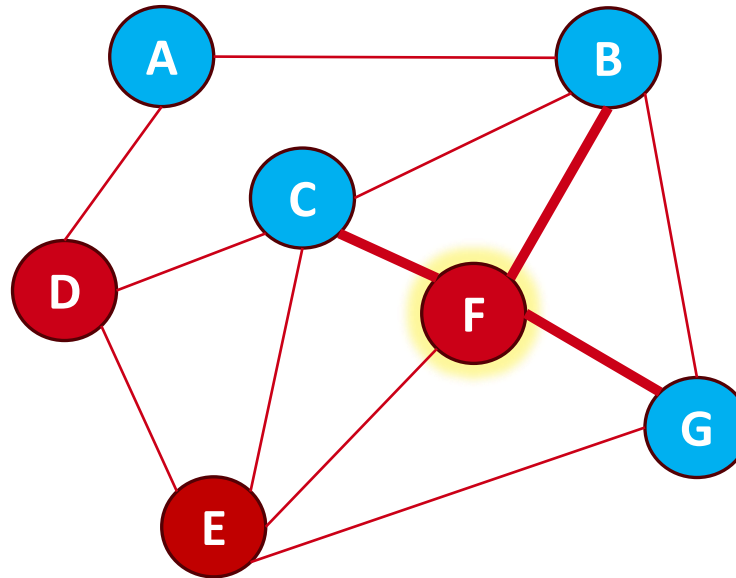
Exchange 1×1



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Algorithmic proposal – Improvement method

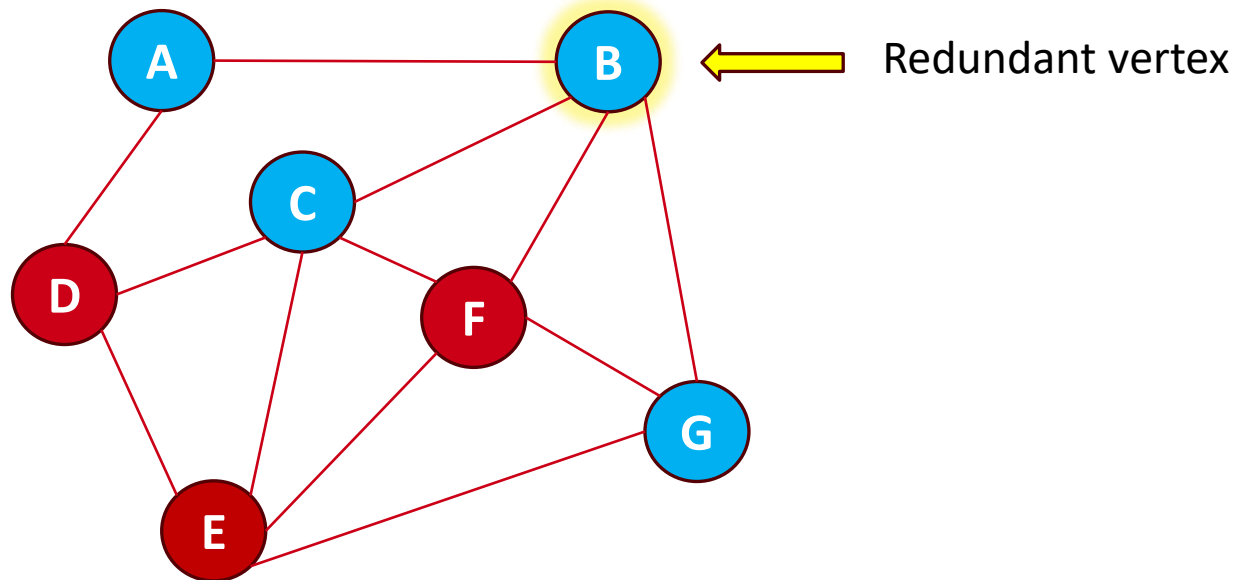
Exchange 1×1



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Algorithmic proposal – Improvement method

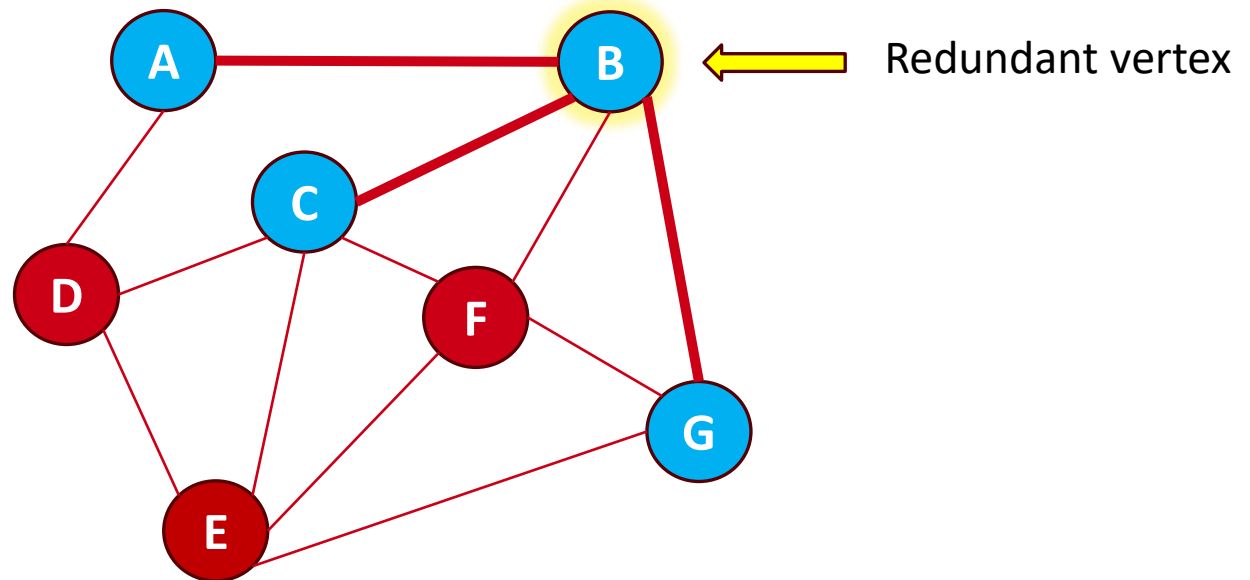
Exchange 1×1



$$k = 2$$

Algorithmic proposal – Improvement method

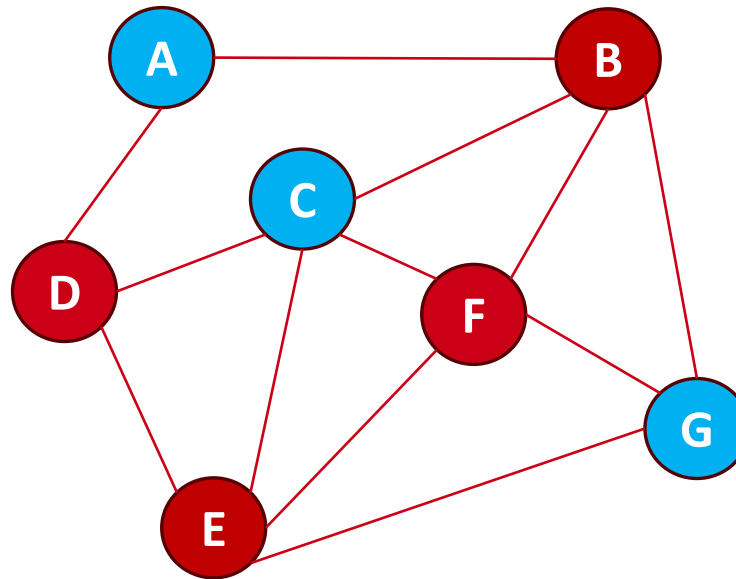
Exchange 1×1



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Algorithmic proposal – Improvement method

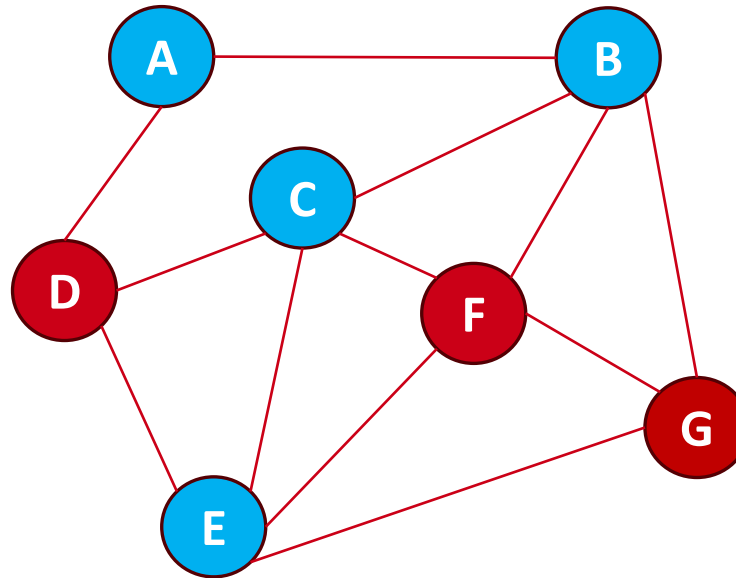
Exchange 1×1



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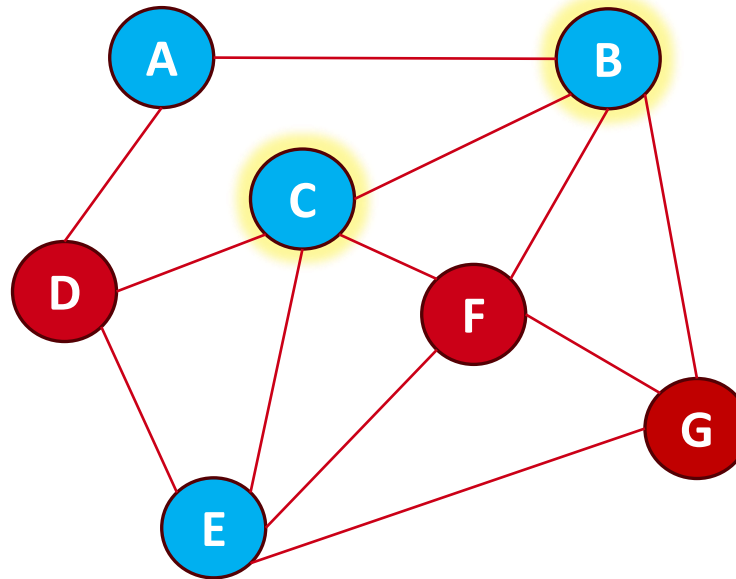
Exchange 2×1



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Algorithmic proposal – Improvement method

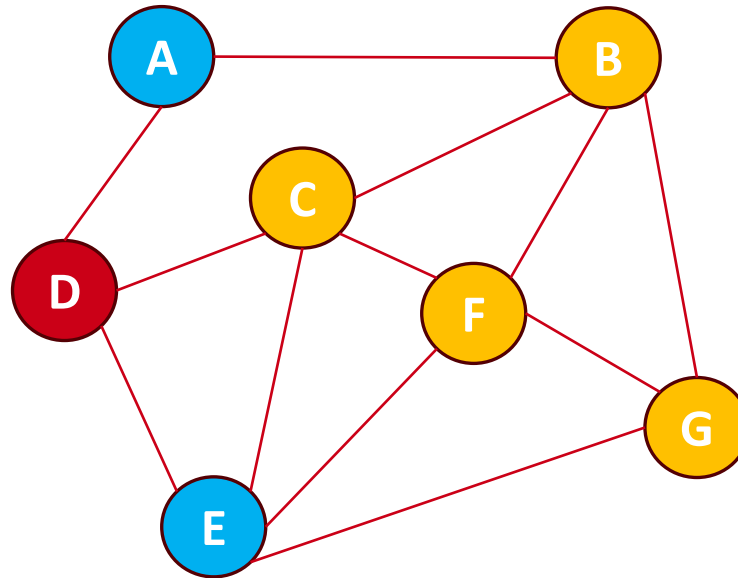
Exchange 2×1



$k = 2$

Algorithmic proposal – Improvement method

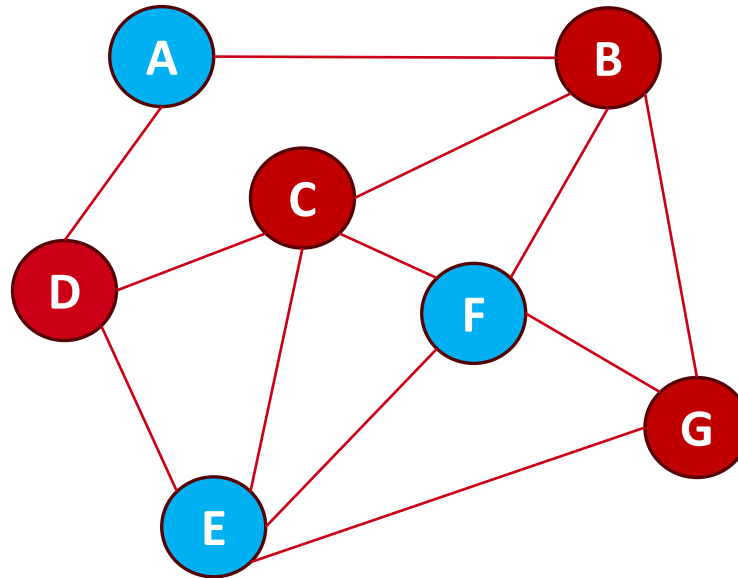
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Algorithmic proposal – Improvement method

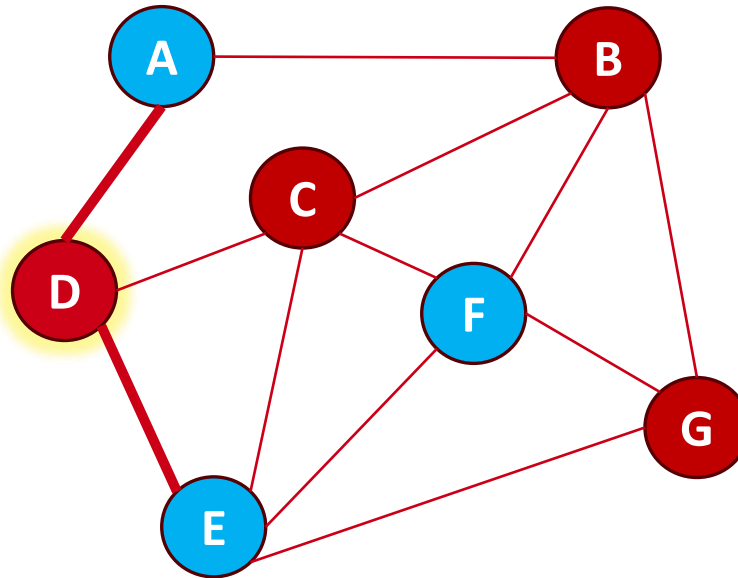
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Algorithmic proposal – Improvement method

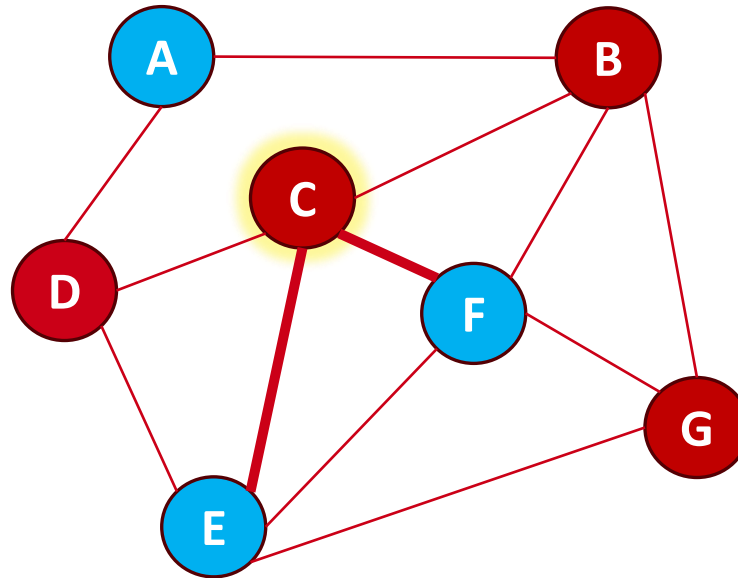
Exchange 2×1



$k = 2$

Algorithmic proposal – Improvement method

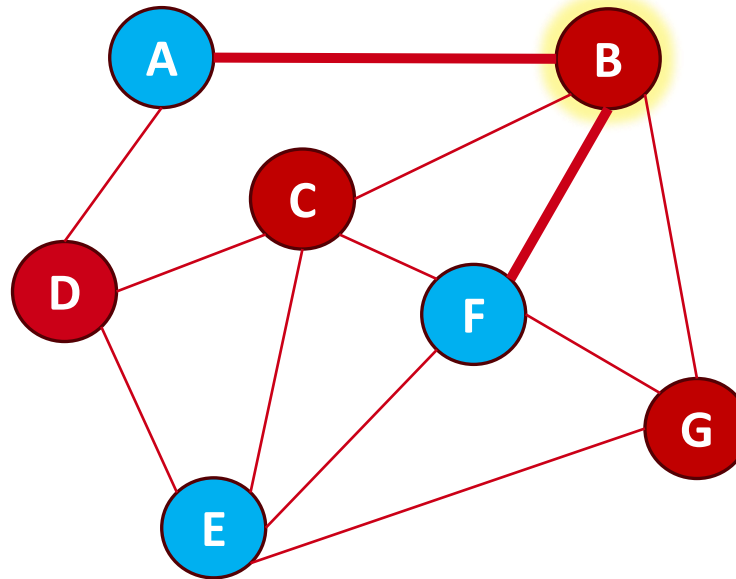
Exchange 2×1



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Algorithmic proposal – Improvement method

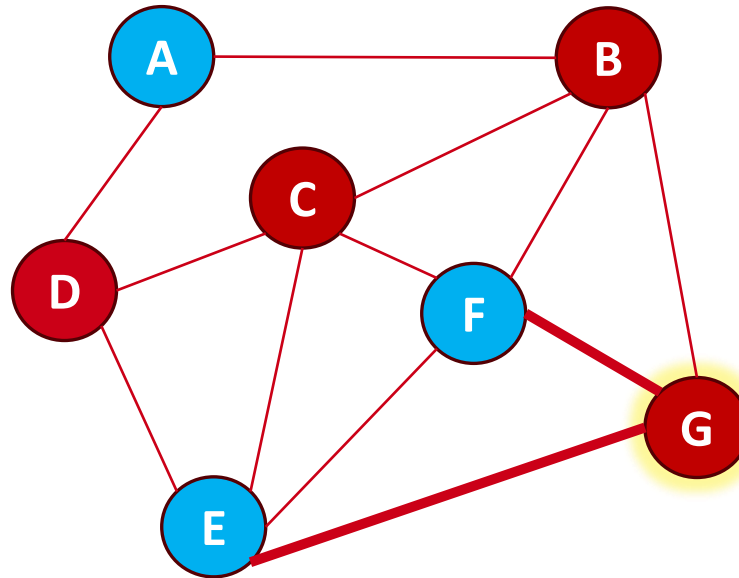
Exchange 2×1



$k = 2$

Algorithmic proposal – Improvement method

Exchange 2×1



$k = 2$

Results

The set of instances **GeneralGraphs (GG)** is formed by:

- 96 instances with $|V| \leq 250$
- 36 instances with $250 < |V| \leq 500$
- 30 instances with $500 < |V| \leq 1000$

Each instance uses three **different k values**:

- $k_{min} = 2$
- $k_{med} = \frac{k_{min} + k_{max}}{2}$
- $k_{max} = \frac{maxDegree}{2}$

Results – Preliminary experimentation

- A preliminar subset of 33 instances with different size was used to determine the **best configuration**

k_{vns}	EX 1 × 1				EX 2 × 1			
	OF	Time (s)	Dev. (%)	#Best	OF	Time (s)	Dev. (%)	#Best
0.10	173.37	0.66	1.89	7	174.70	2.44	2.76	4
0.20	172.77	2.96	1.30	11	173.10	9.33	1.89	8
0.30	172.77	6.29	1.44	13	172.90	20.32	1.45	9
0.40	172.37	10.67	1.23	14	173.13	34.80	1.47	9
0.50	172.17	15.67	1.11	17	173.13	47.59	1.45	9

Results – Final experimentation

- With the **best parameters**:
 - Exchange 1×1
 - $k_{vms} = 0.5$
 - Stopping criterium \rightarrow 4000 iterations

	OF	Time (s)	Dev. (%)	#Best
VNS	162.78	56.62	0.71	114
RLS_RCC	161.68	58.21	0.00	159

Results – Final experimentation

Instance	VNS				RLS_RCC			
	OF	Time (s)	Dev. (%)	#Best	OF	Time (s)	Dev. (%)	#Best
k_{min}	88.72	79.43	1.51	31	87.35	34.66	0.00	53
k_{med}	165.06	74.54	0.48	38	163.69	81.32	0.00	53
k_{max}	234.57	15.89	0.13	45	234.00	58.66	0.00	53

Conclusions & Future work

- The proposal:
 - Obtains **promising results**
 - **Scales better as parameter k increases**
- Future work:
 - Consider **new diversity enhancing components**
 - Analyze **algorithm configuration** in detail
 - Consider new neighborhoods to **increase the space search**

Acknowledgments

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- ❑ **CIRMA**: This work has been supported by Comunidad Autónoma de Madrid, CIRMA-CM Project (TEC-2024/COM-404).
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