Ordered Median Tree Location Problem (OMT)

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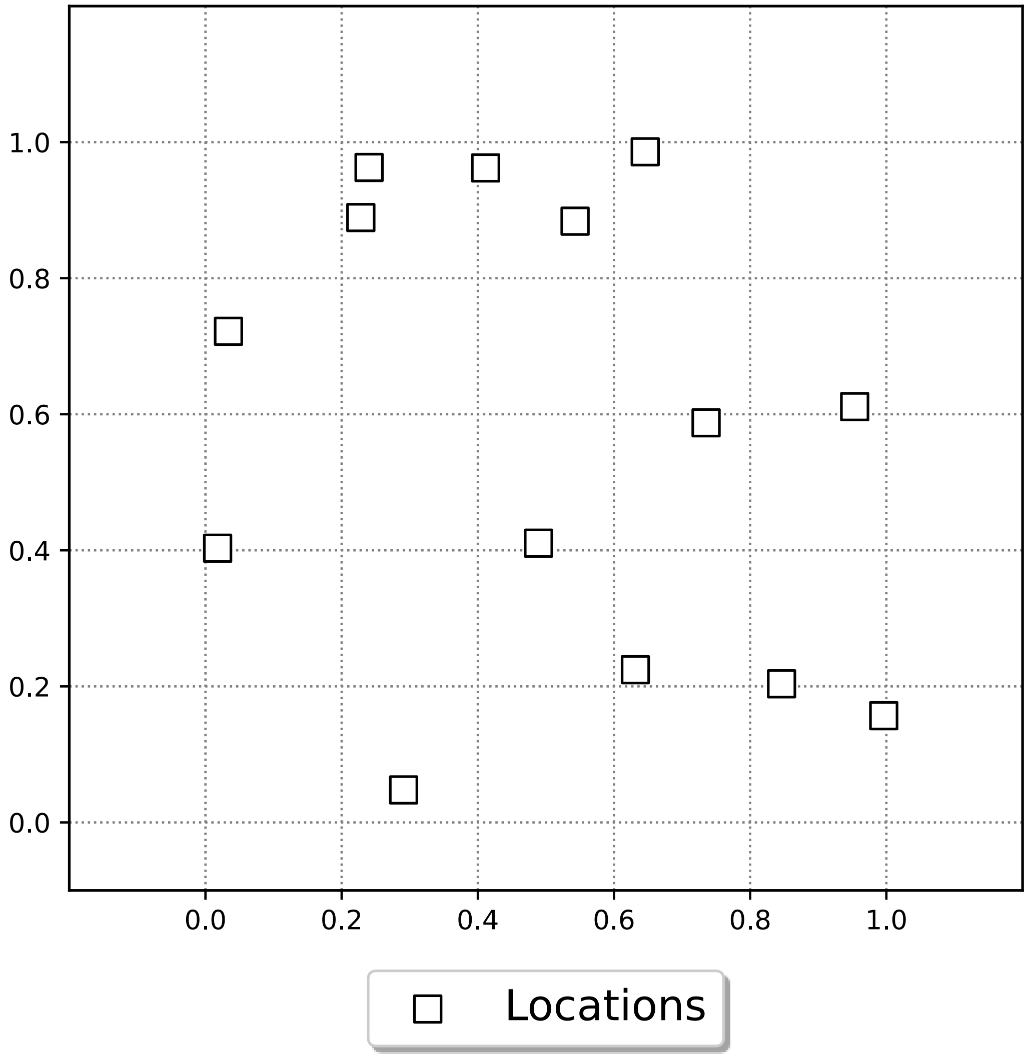
Outline

1. Introduction
2. Proposal
3. Results
4. Conclusions and future work

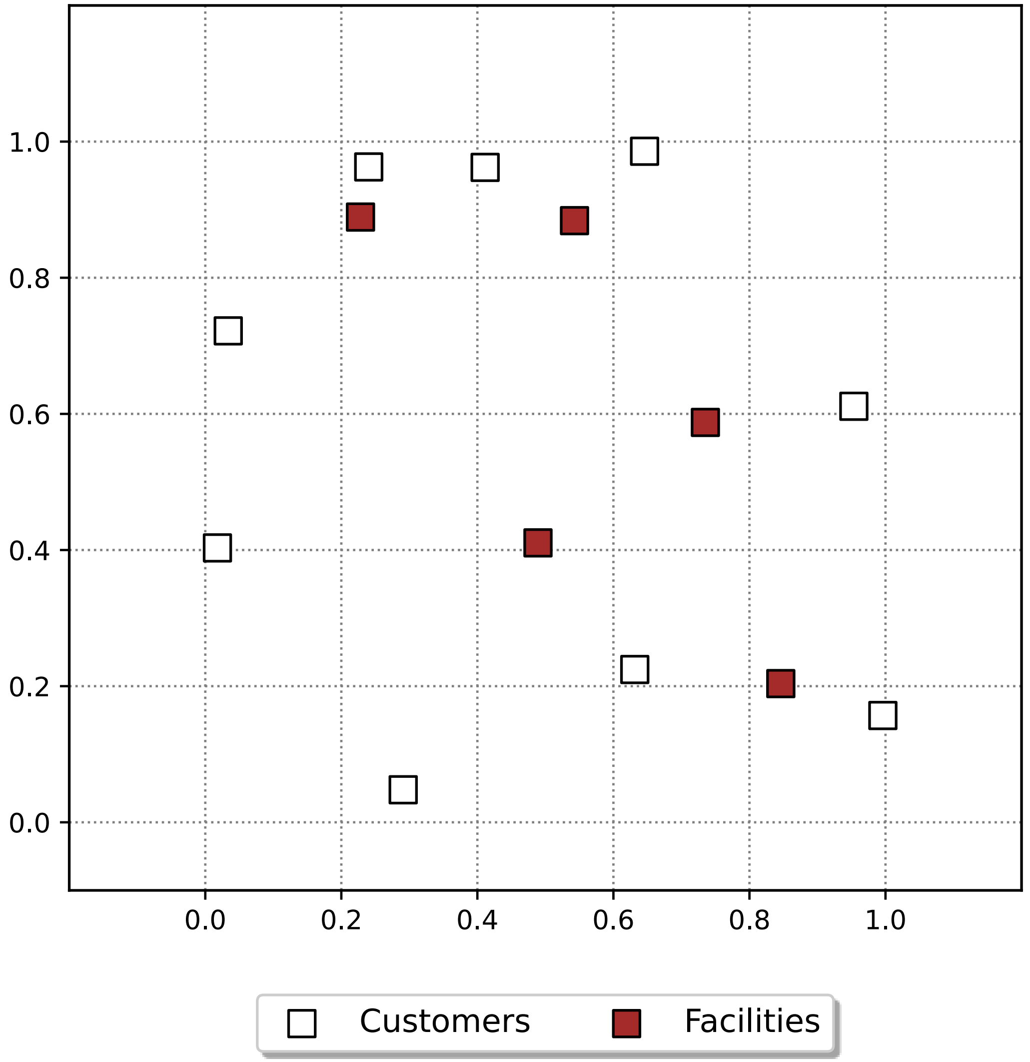
Introduction

- The Ordered Median Tree Location Problem (**OMT**) belongs to the family of **facility location problems**.
- The objective is to **minimize** the sum of the costs of the **tree edges** connecting the facilities, plus the cost of **assigning customers** in a weighted manner.
- Applications:
  - Network configuration, telecommunications.
  - Service planning, transportation, logistics.

Introduction



Introduction



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  - Open **p** facilities.

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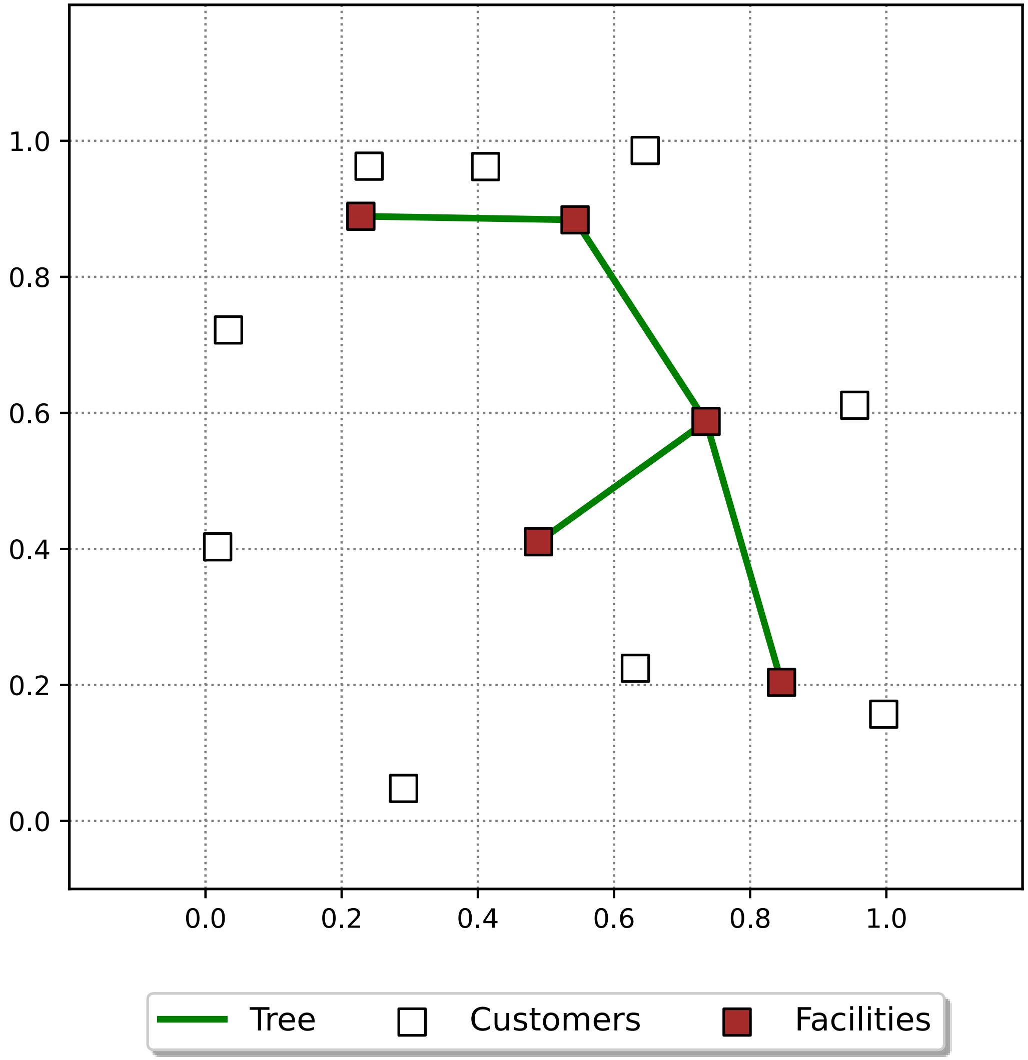

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  - Open **p** facilities.
  - Connect them using an undirected **tree**.

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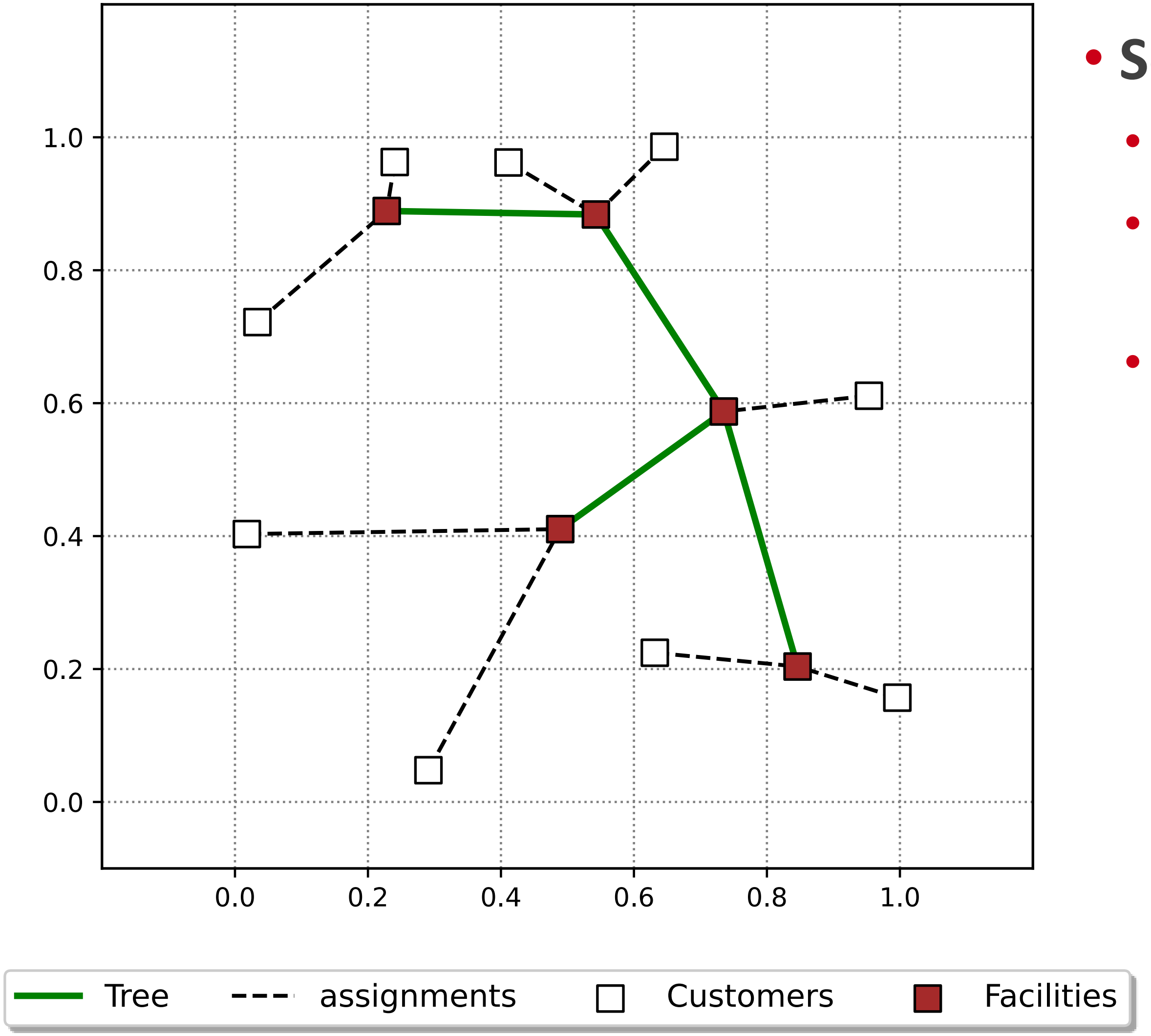

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  - Open **p** facilities.
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  - Assign each **customer** to an open facility.

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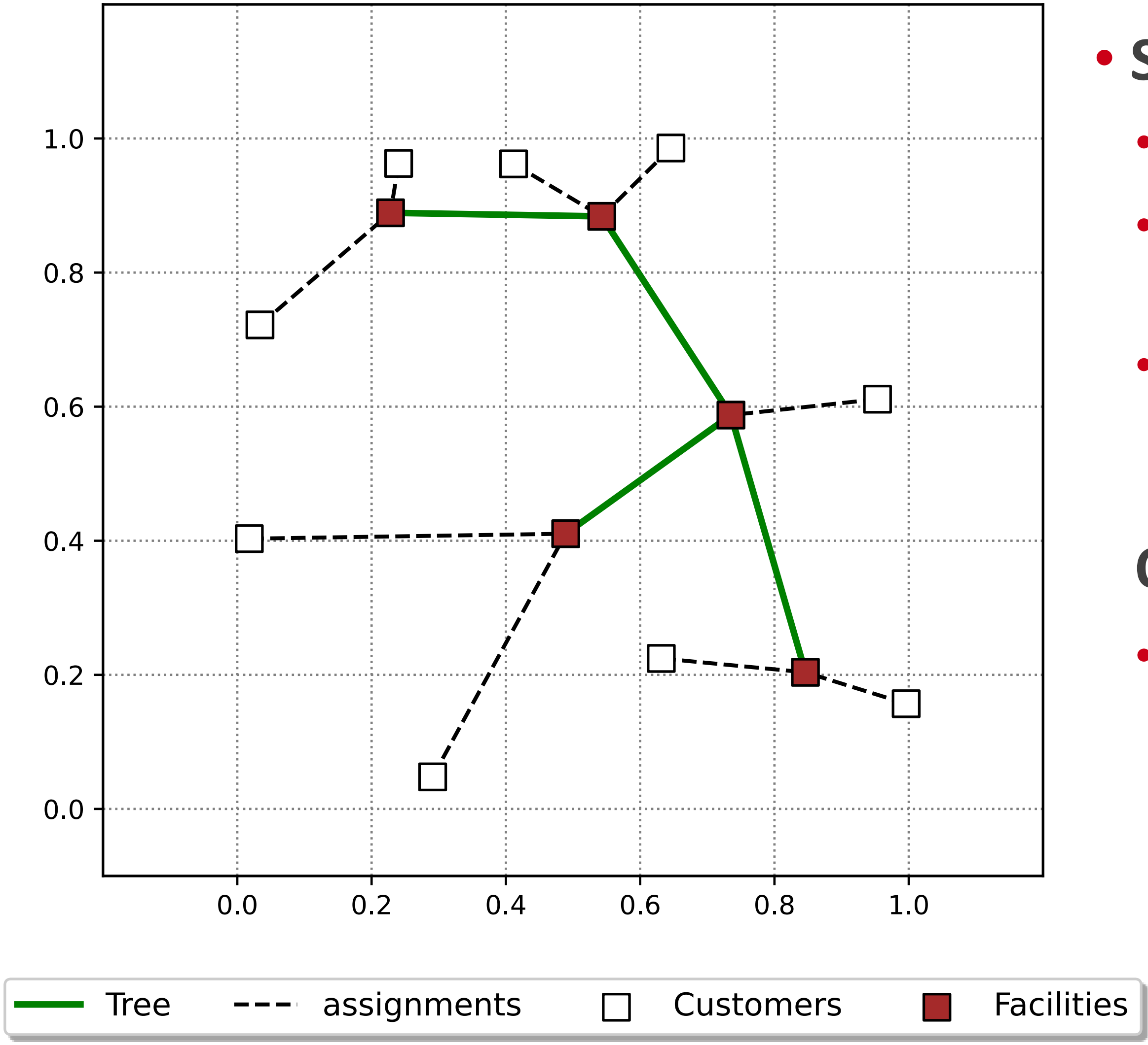

- **Solution:**
  - Open **p** facilities.
  - Connect them using an undirected **tree**.
  - Assign each **customer** to an open facility.

Objective:

- Minimize the ordered **weighted** sum of assignment costs (tree and customers).

Formulation

$$\text{Minimize} \quad \frac{1}{\sum_{\ell \in V} \lambda_\ell} \sum_{\ell \in V} \sum_{(i,j) \in A} \lambda_\ell c_{ij} x_{ij}^\ell + \frac{1}{p-1} \sum_{(i,j) \in E} c_{ij} z_{ij} \tag{1}$$

$$\text{Subject to} \quad \sum_{i \in V} x_{ii} = p \tag{2a}$$

$$\sum_{j \in V} x_{ij} = 1 \qquad i \in V \tag{2b}$$

$$x_{ij} \leq x_{jj} \qquad (i,j) \in A : i \neq j \tag{2c}$$

$$2 z_{ij} \leq x_{ii} + x_{jj} \qquad (i,j) \in E \tag{3a}$$

$$z_{ij} \in \mathcal{T} \qquad (i,j) \in E \tag{3b}$$

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$$x_{ij}, x_{ij}^\ell, z_{pq} \in \{0,1\} \qquad (i,j) \in A, \ell \in V, (p,q) \in E \tag{5}$$

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Pozo, M.A., Puerto, J., Torrejón, A., 2024. The Ordered Median Tree Location Problem. Computers & Operations Research 169, 106746. https://doi.org/10.1016/j.cor.2024.106746

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GUROBI OPTIMIZATION

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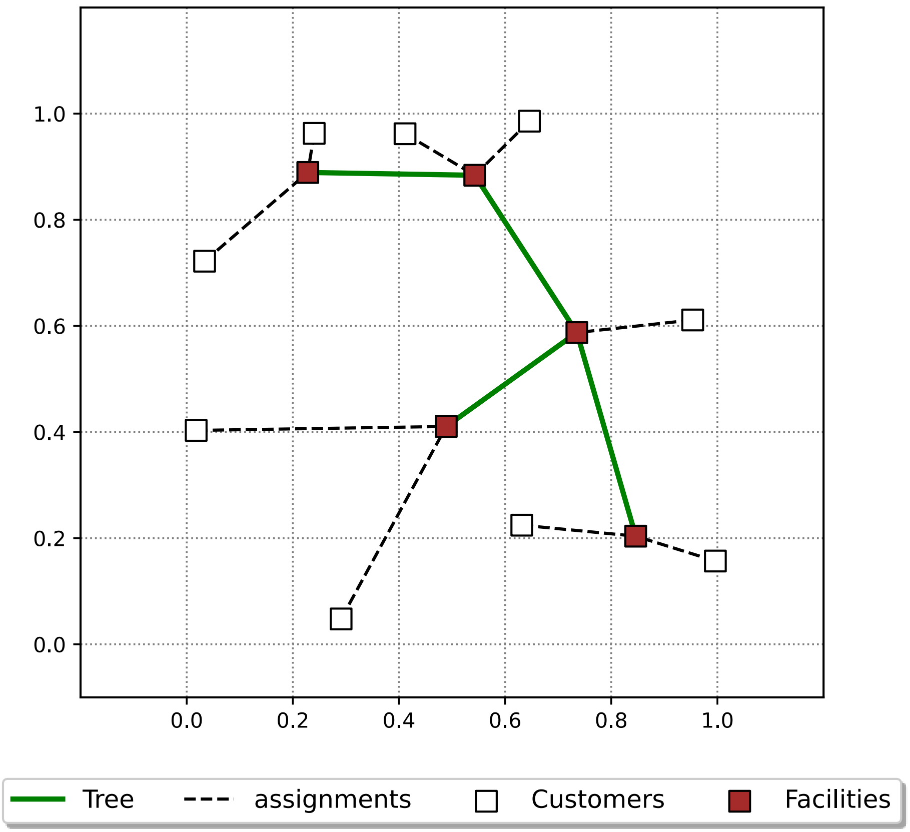
Cost ordering



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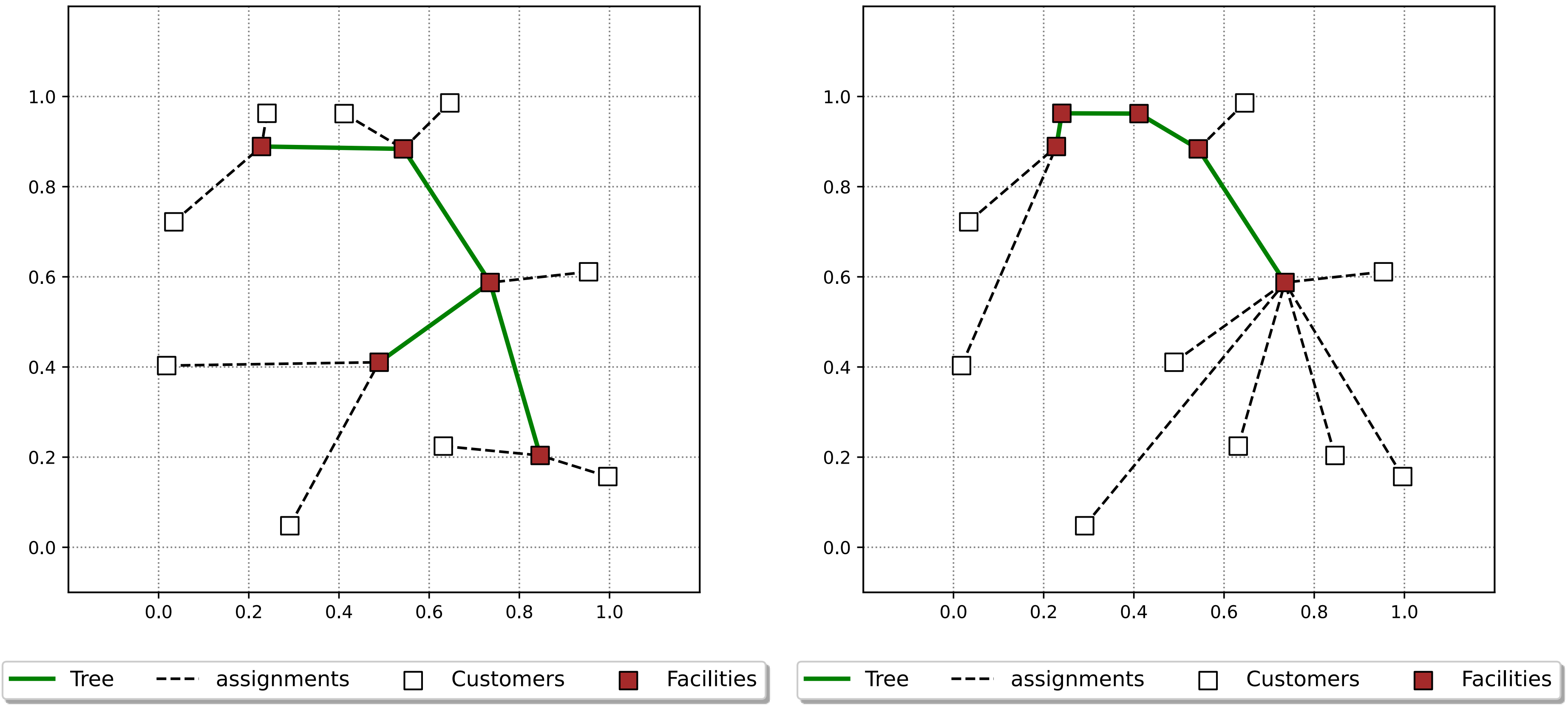
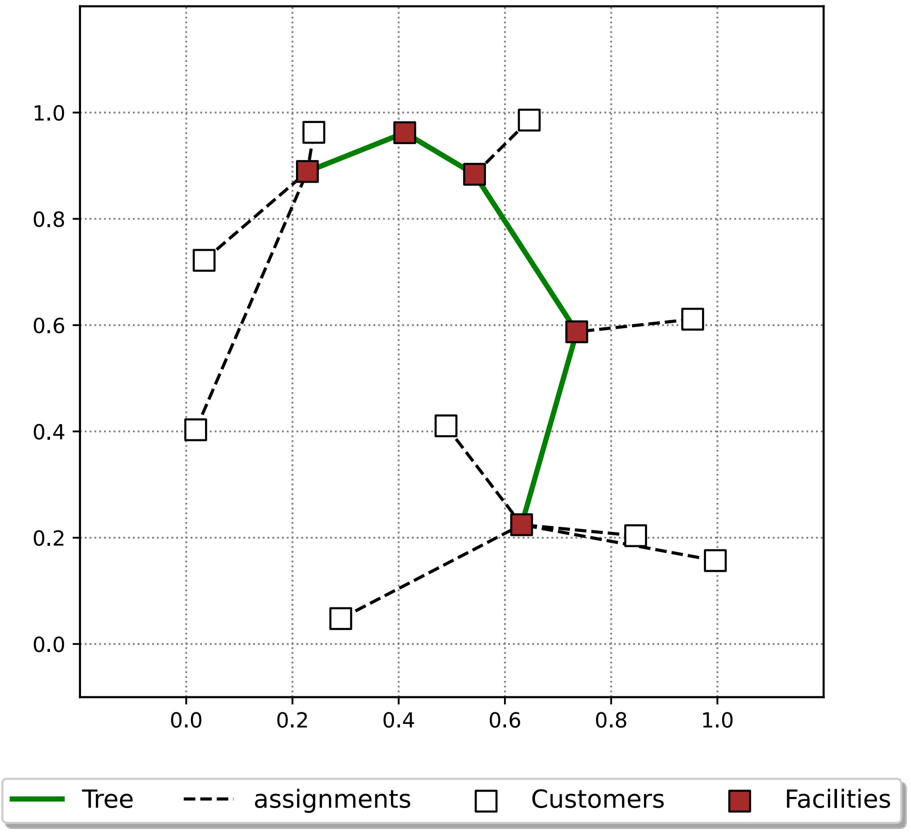


$$\lambda = (1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1)$$

Median criterion

Cost ordering



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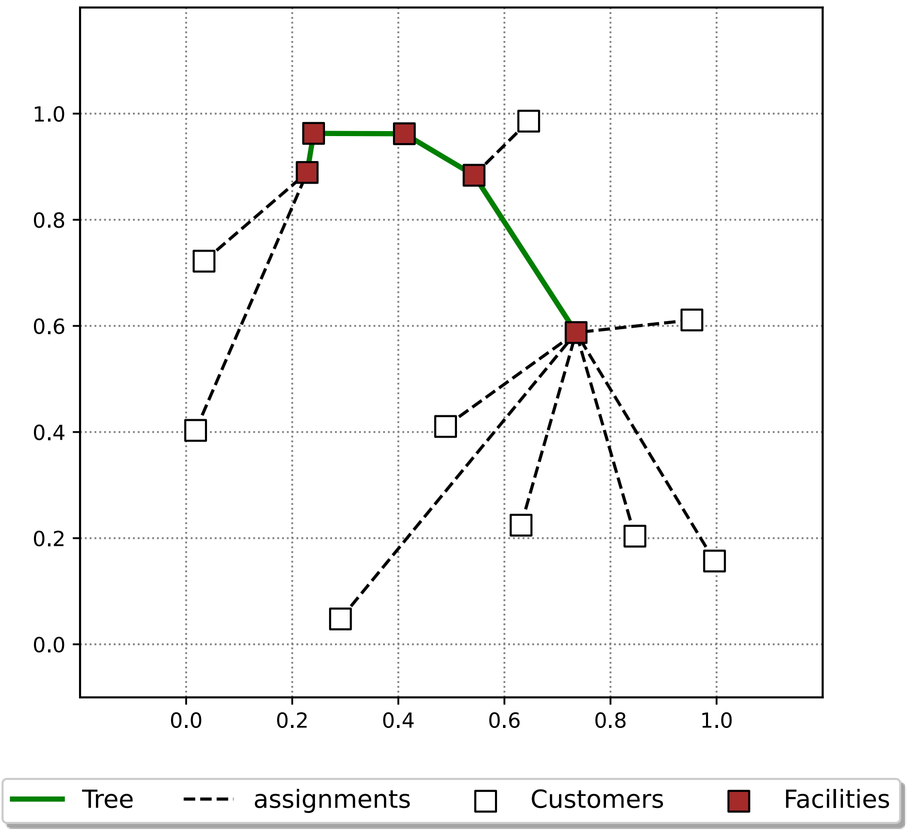
K-centrum criterion



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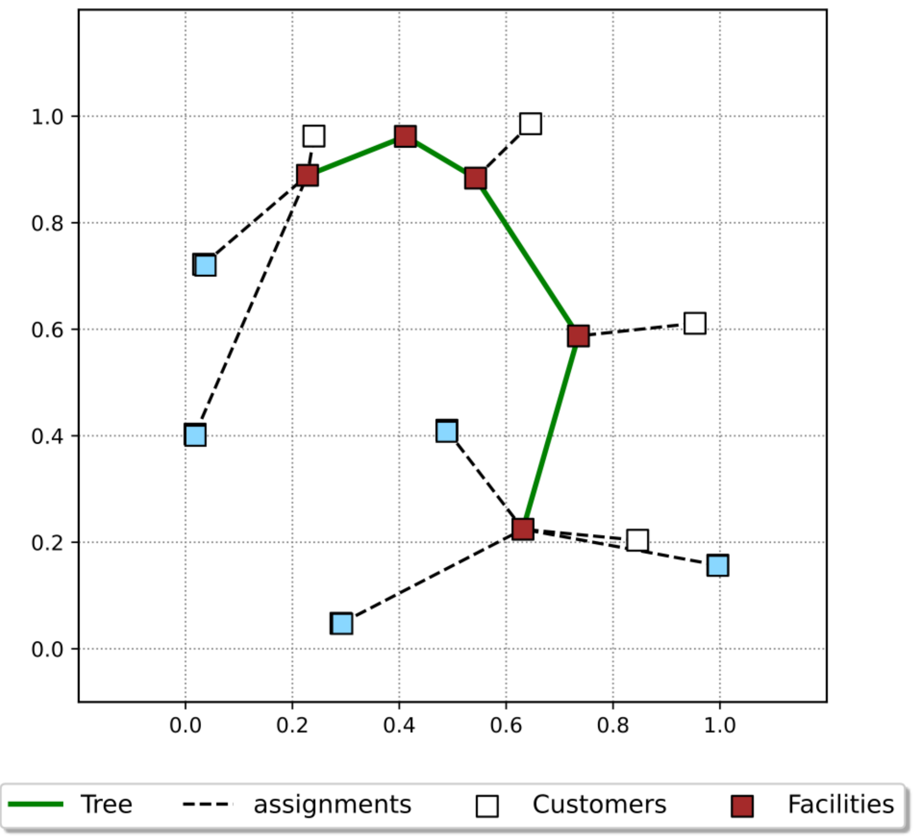

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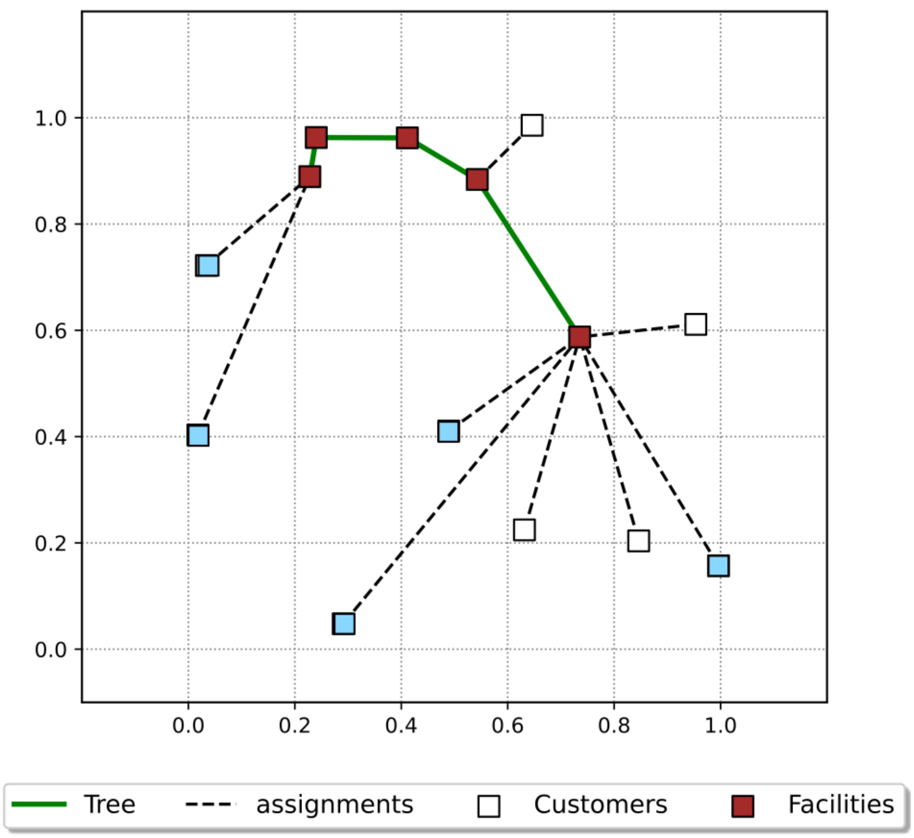

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Median criterion

Search Space Reduction

- $F1_u^{mtz}$: $$x_{ij}, x_{ij}^{\ell}, z_{pq} \in \{0,1\} \qquad (i,j) \in A, \ell \in V, (p,q) \in E \qquad (5)$$

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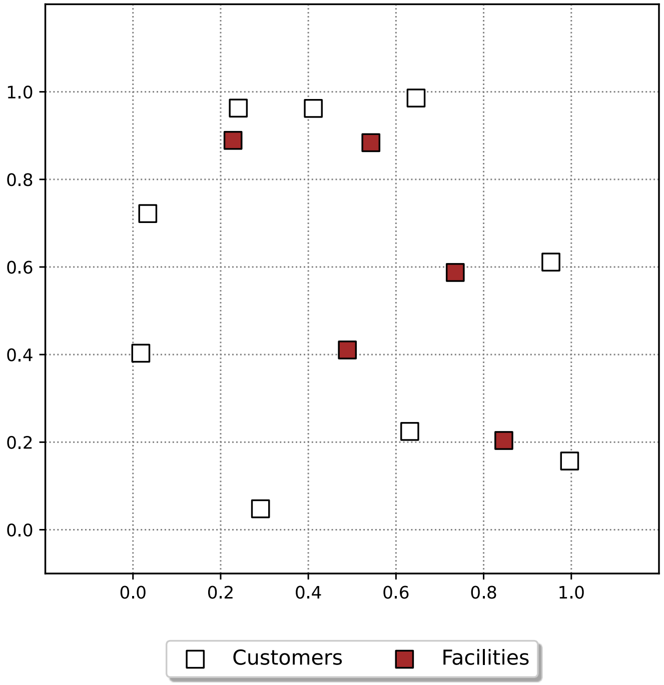

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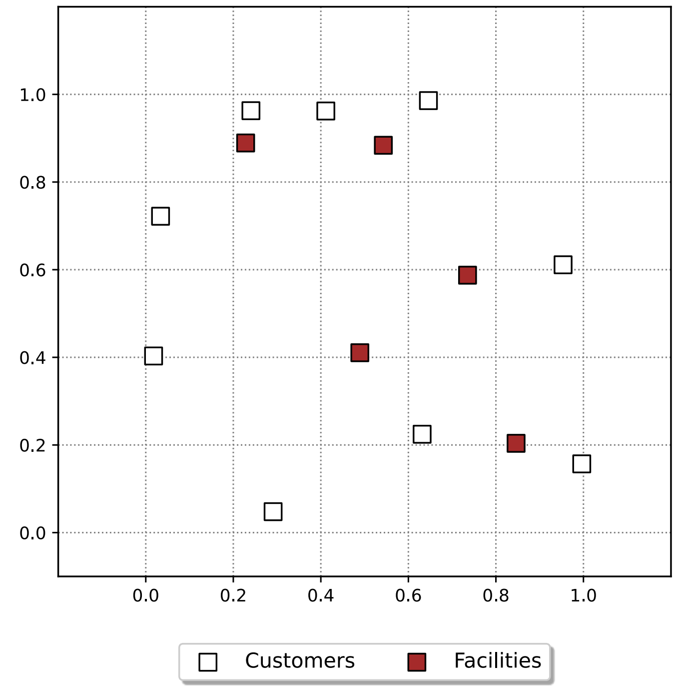
- Optimal customer assignment

- Minimum Spanning Tree (MST)



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  - Assign each customer to the **closest open facility**

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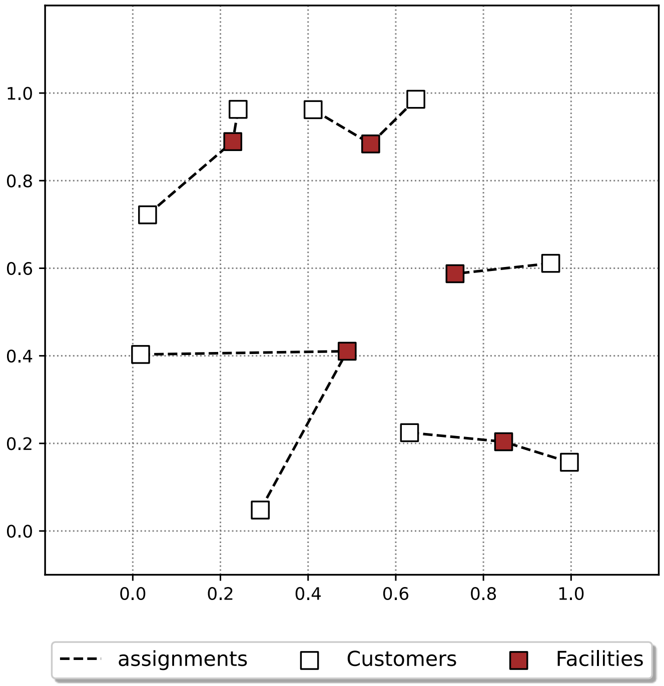

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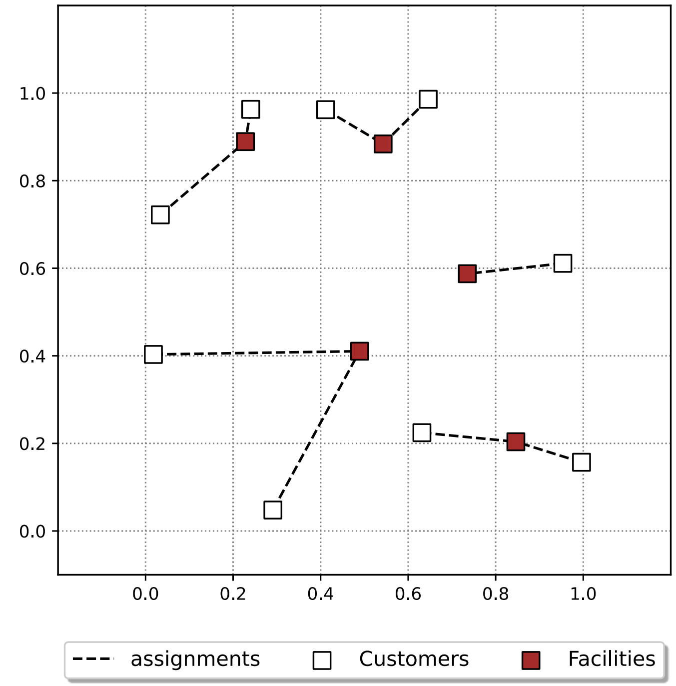

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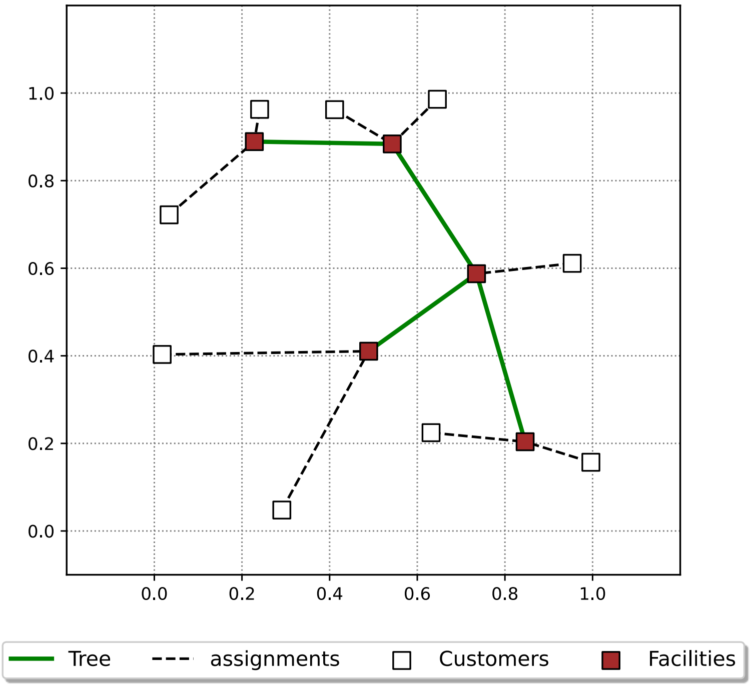
- Minimum Spanning Tree (MST)
  - **Prim's** Algorithm ➔ $O(p^2)$



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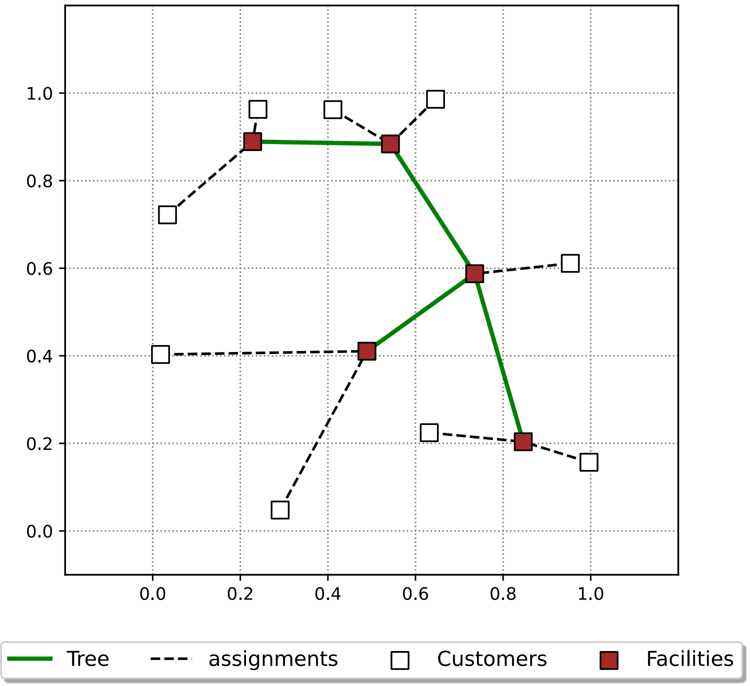
- Minimum Spanning Tree (MST)
  - **Prim's** Algorithm ➔ $\mathbf{O(p^2)}$

- Combinatorial search space
  - **Facility selection**, explored by the metaheuristic proposal.



Minimum Spanning Tree (MST)

Prim's Algorithm

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Prim's Algorithm

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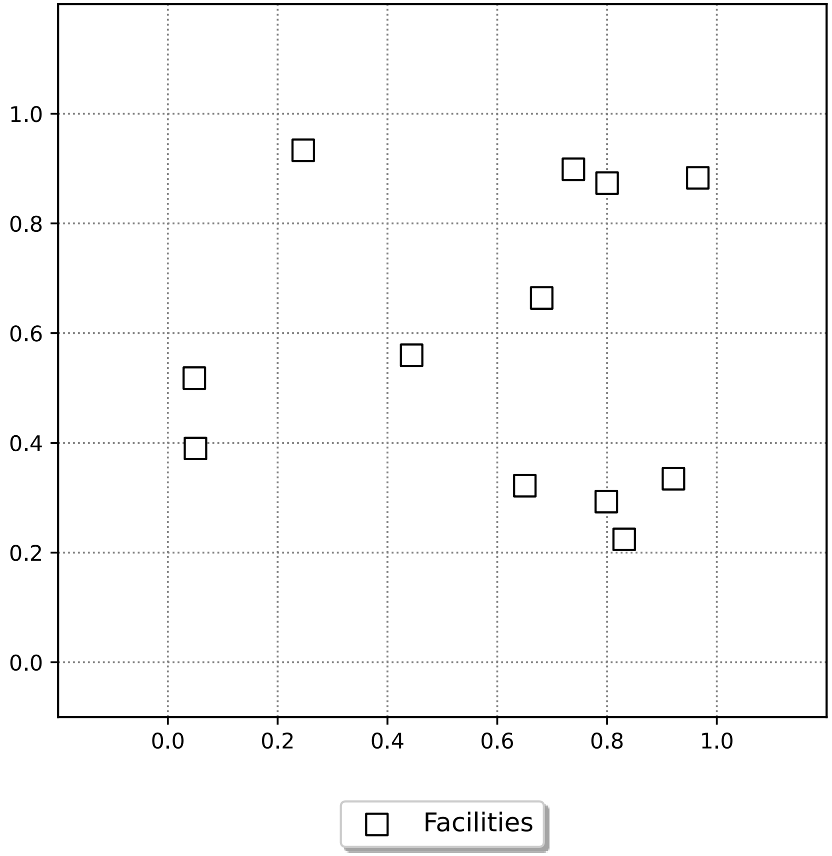

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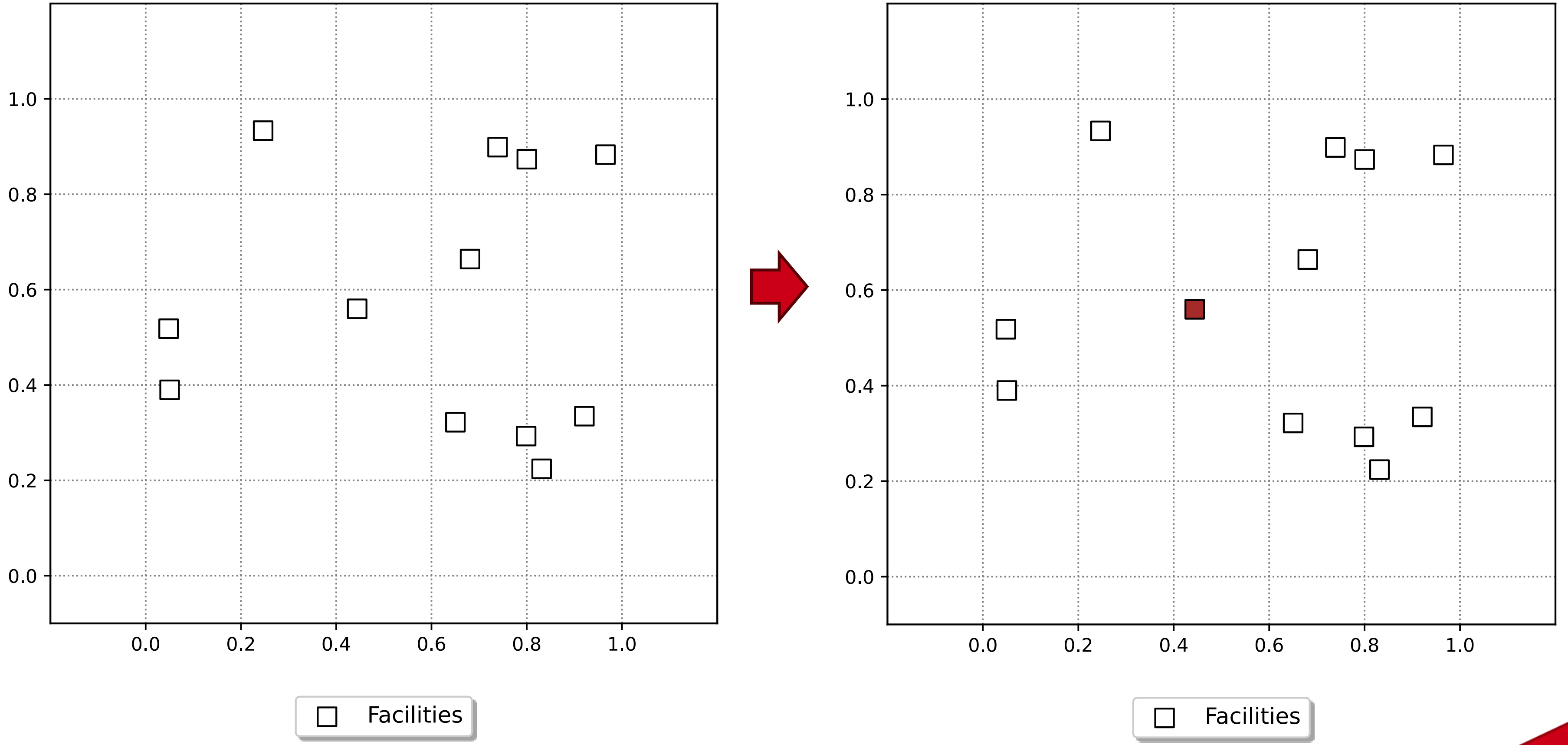

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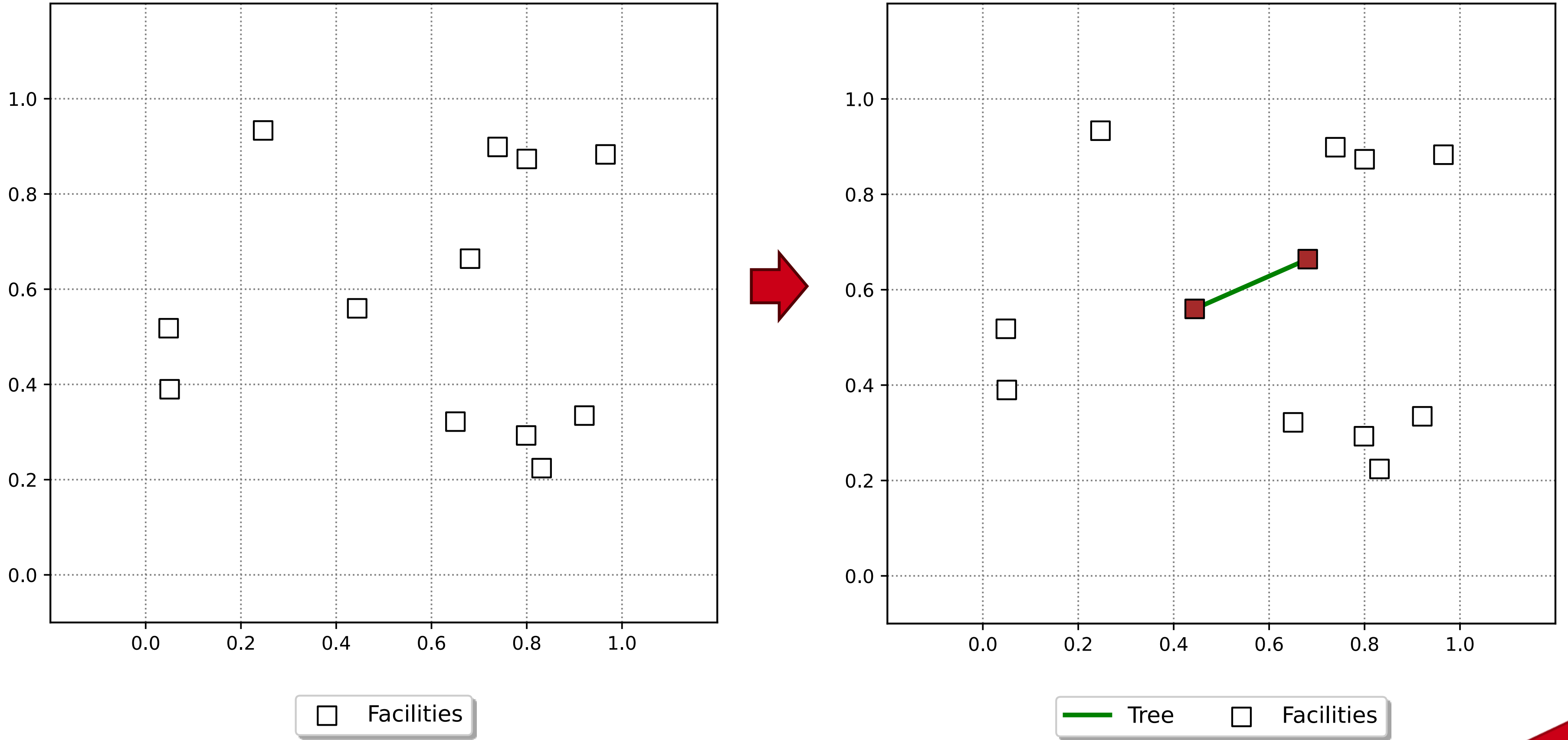
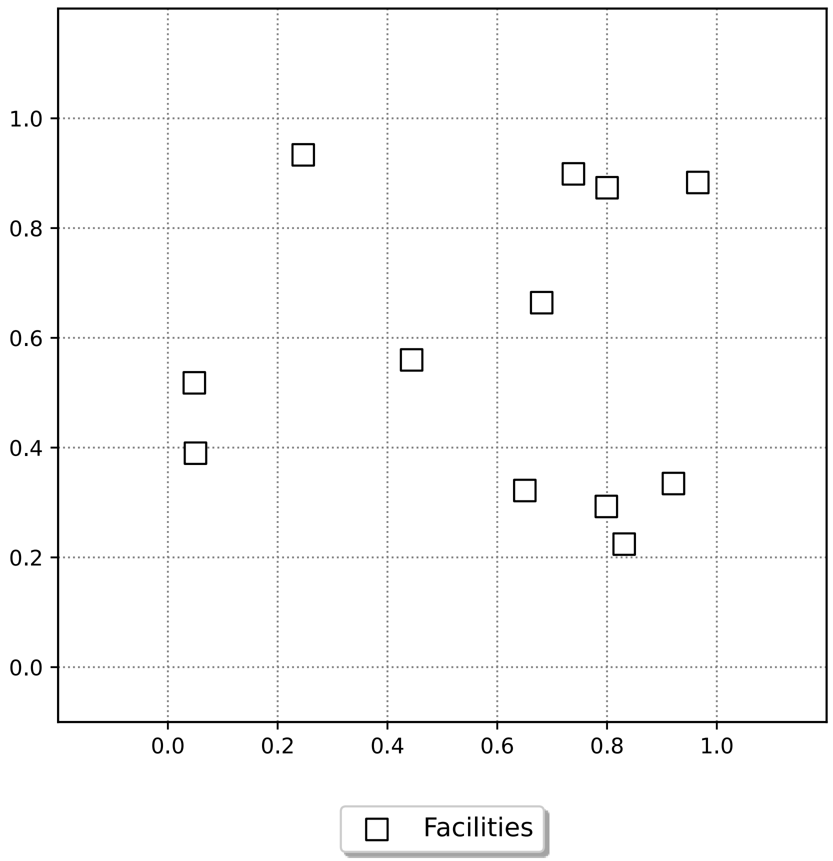
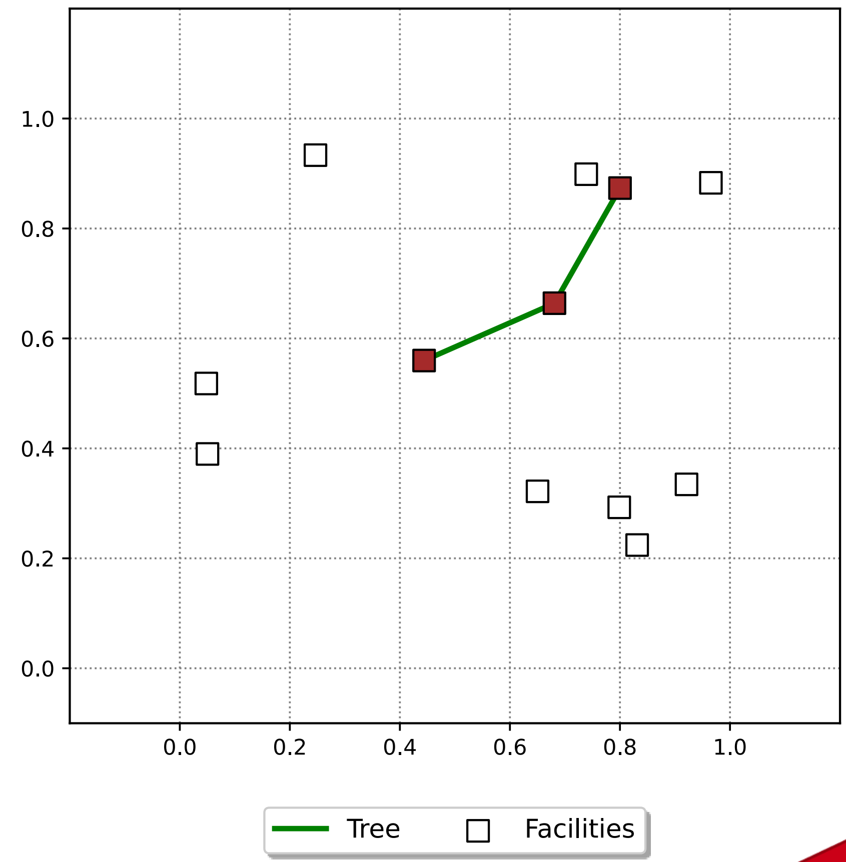


Minimum Spanning Tree (MST)

Prim's Algorithm

- Complete graph ➔ $O(p^2)$





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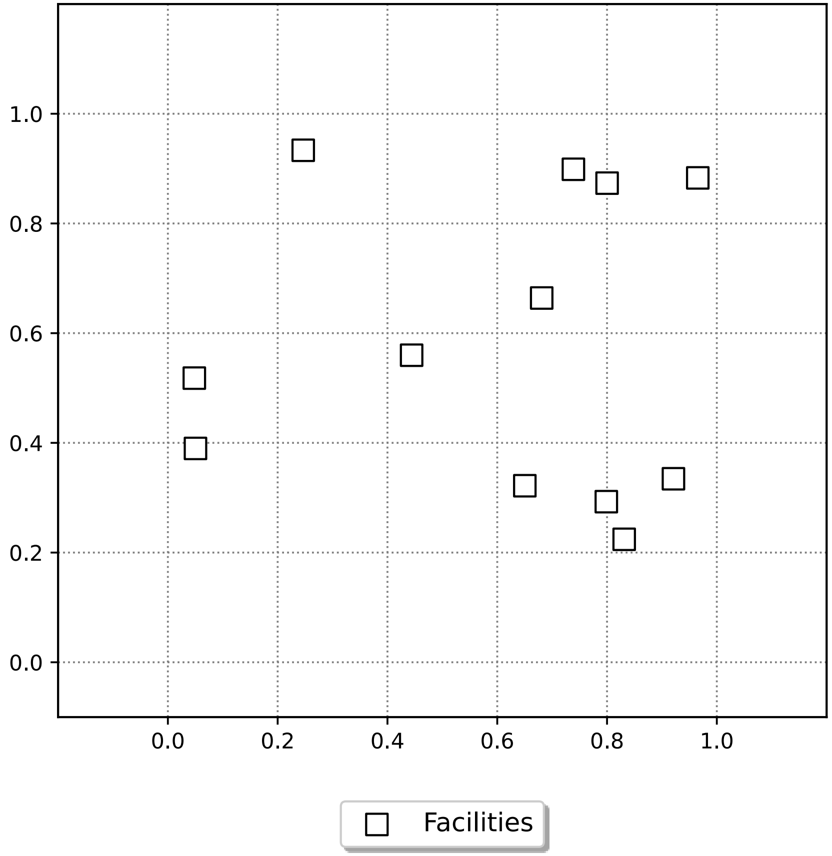

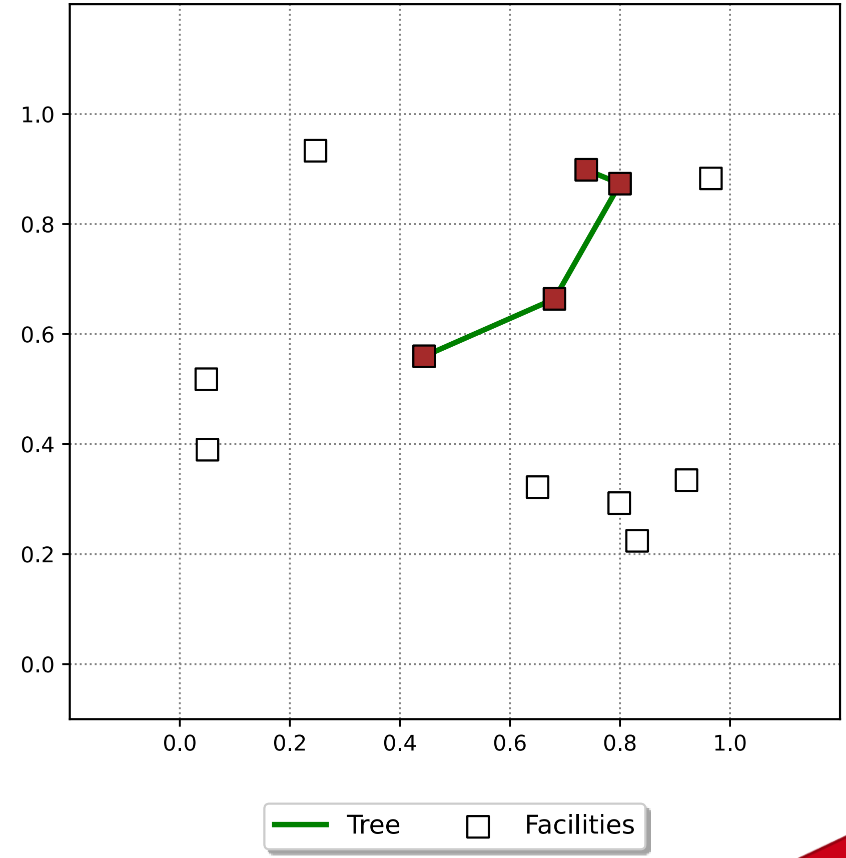

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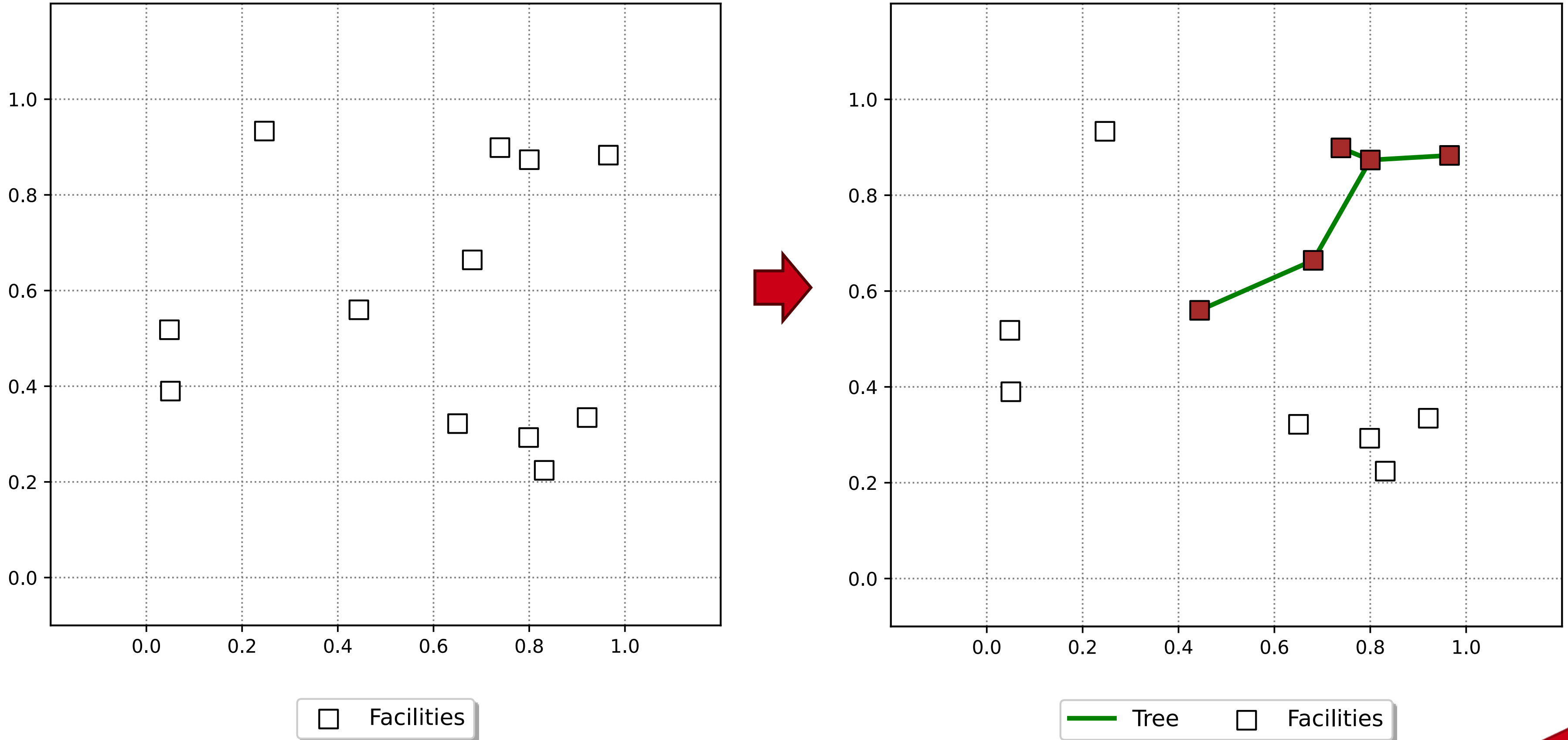

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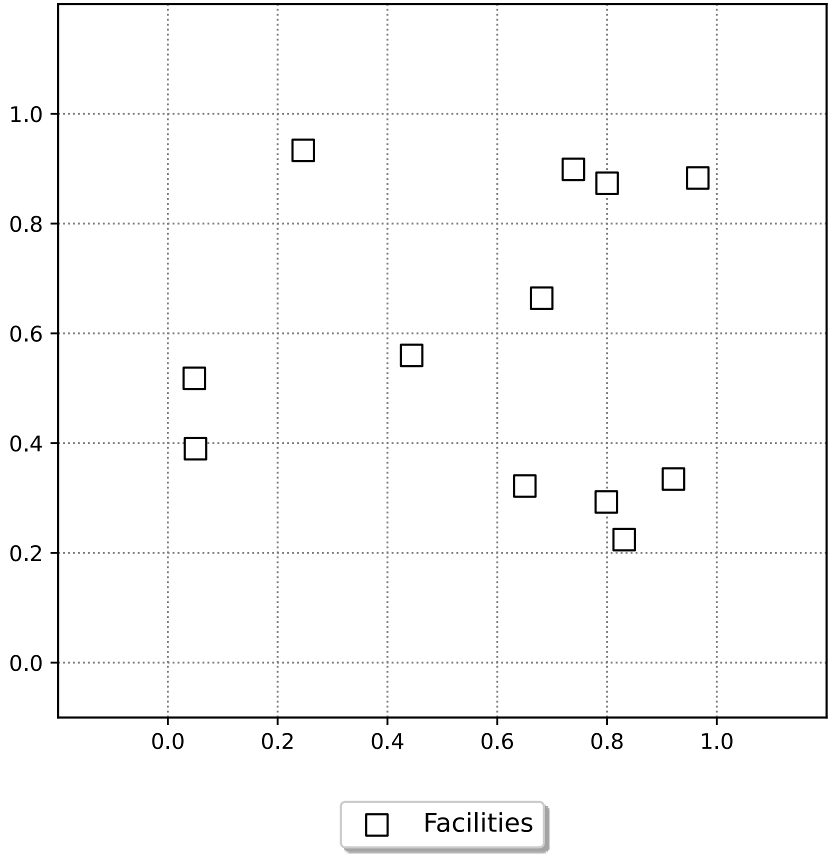

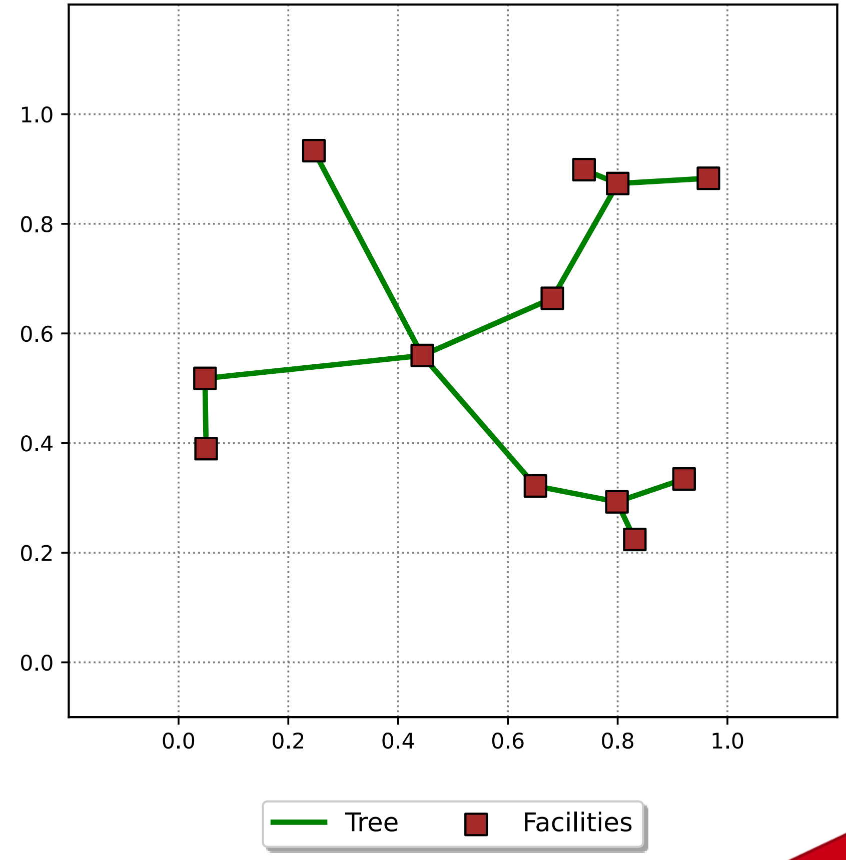

Proposal

- The **Greedy Randomized Adaptive Search Procedure** (GRASP) is a multi-start metaheuristic proposed in 1989 by T. A. Feo and M. G. C. Resende.

```
Algorithm GRASP (α, P, N):
    S* ← ∅
    for i = 1...N do
        S ← Construction(α, P)
        S ← LocalSearch(S)
        if F(S) < F(S*) then
            S* ← S
    return S*
```

Feo, T.A., Resende, M.G.C., 1989. A probabilistic heuristic for a computationally difficult set covering problem. Operations Research Letters 8, 67–71. https://doi.org/10.1016/0167-6377(89)90002-3

Proposal

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Algorithm *GRASP* (α, P, N)**:**

    $S^* \leftarrow \emptyset$

    for $i = 1...N$ **do**

        $S \leftarrow \texttt{Construction}(\alpha, P)$

        $S \leftarrow \texttt{LocalSearch}(S)$

        if $\mathcal{F}(S) < \mathcal{F}(S^*)$ **then**

            $S^* \leftarrow S$

    return S^*

- The customer assignments and MST are obtained **efficiently**.

Feo, T.A., Resende, M.G.C., 1989. A probabilistic heuristic for a computationally difficult set covering problem. Operations Research Letters 8, 67–71. https://doi.org/10.1016/0167-6377(89)90002-3

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- The customer assignments and MST are obtained **efficiently**.

- **GRASP** focuses only on **facility selection**.

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Proposal

- Objective-function-based construction heuristic $\mathcal{F}'$

$$\mathcal{F}(\phi, I) = \frac{1}{\sum_{\ell \in V} \lambda_\ell} \sum_{\ell \in V} \sum_{(i,j) \in A} \lambda_\ell c_{ij} x_{ij}^\ell + \frac{1}{p-1} \sum_{(i,j) \in E} c_{ij} z_{ij} \qquad (1)$$

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$$\mathcal{F}'(\phi, I, u) = \frac{1}{\sum_{\ell \in V} \lambda_\ell} \sum_{\ell \in V} \sum_{(i,j) \in A} \lambda_\ell c_{ij} x_{ij}^\ell + \min_{v \in \mathcal{G}^{-1}(\phi) \setminus u} c_{uv} \quad (2)$$

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Results

- **Programming languages**: Java 25, Python 3.8, and Gurobi 11.
- **Server specifications**: AMD EPYC 7282 limited to 32 virtual CPU cores and 32 GB of RAM.
- **Instances**: 105 Median, 105 k-center, 105 k-trimmed (20–100 nodes, 5–50 **p** facilities).
- **Metrics**:
  - $\mathcal{F}$: value of the objective function.
  - **Time (s):** execution time in seconds.
  - **Dev (%)**: deviation from the best value found.
  - **# Best**: total number of instances in which the algorithm obtains the best solution in the experiment.

Results

Table 1: Comparison between the exact formulation $F1_u^{mtz}$ and GRASP.

Criterion	Algorithm	$\mathcal{F}$	Time (s)	Dev (%)	# Best
Median	$F1_u^{mtz}$	4,985.91	1,114.94	0.029798	129
	GRASP	**4,984.93**	**3.97**	**0.000000**	**135**
k-Centrum	$F1_u^{mtz}$	8,706.09*	2,559.81	0.644954*	93
	GRASP	**8,631.98**	**4.26**	**0.000000**	**135**
k-Trimmed Mean	$F1_u^{mtz}$	3,531.95*	1,388.04	0.054605*	123
	GRASP	**3,520.00**	**4.05**	**0.000000**	**135**
Aggregated	$F1_u^{mtz}$	5,732.06*	1,687.59	0.241589*	345
	GRASP	**5,712.30**	**4.09**	**0.000000**	**405**

Conclusions and future work

- **GRASP** identifies the **best solutions** from the set of instances.
- The metaheuristic approach improves the **performance** and reduces **computation time** compared to models.

- Expand the **dataset**.
- Explore **machine learning** methods (GNN) to guide the search.
- Implement dynamic **MST updates**.
- Enhance the GRASP **improvement phase.**

Acknowledgments

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- **Ministerio de Ciencia e Innovación:** ref. PID2021-126605NB-I00, PID2021-125709OA-C22 and PID2024-156045NB-I00 financed by MCIN /AEI /10.13039/501100011033 y por ERDF A way of making Europe, y ref. FPU24/01134.
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AGENCIA ESTATAL DE INVESTIGACIÓN

FPU24/01134

Ordered Median Tree Location Problem (OMT)

FPU24/01134

16th Metaheuristics International Conference

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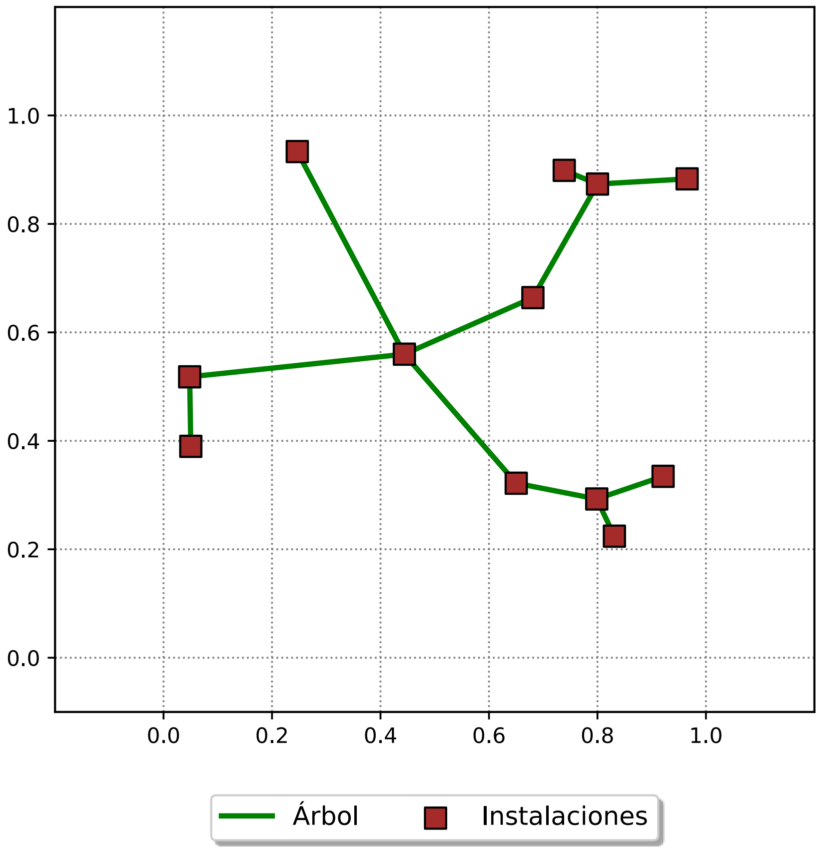
Dynamic MST updates

1) Leaf node



Dynamic MST updates

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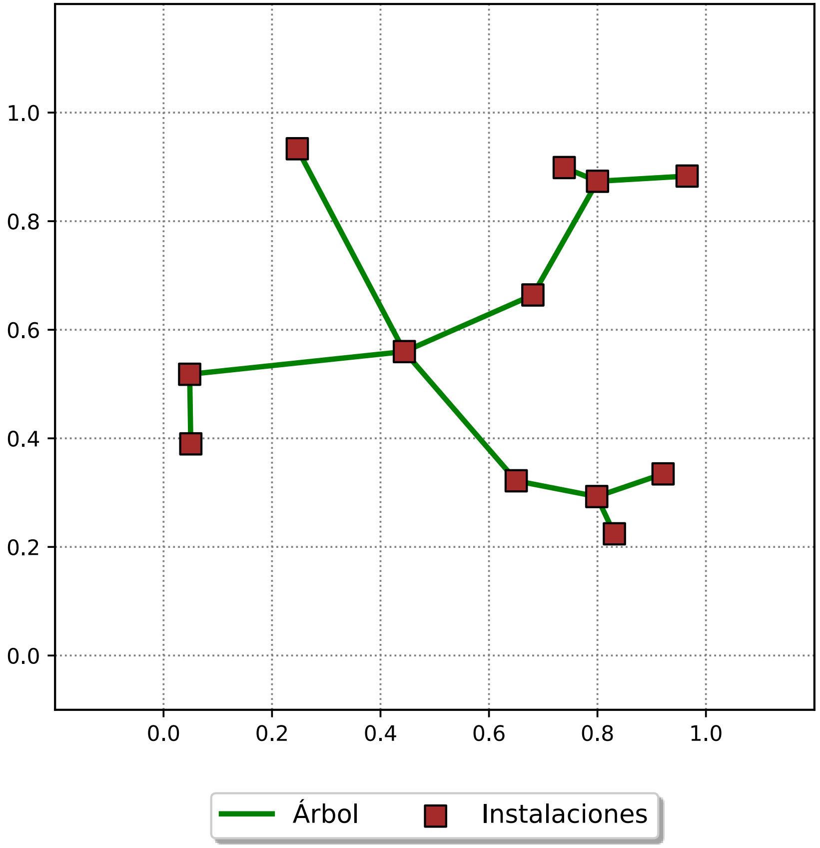
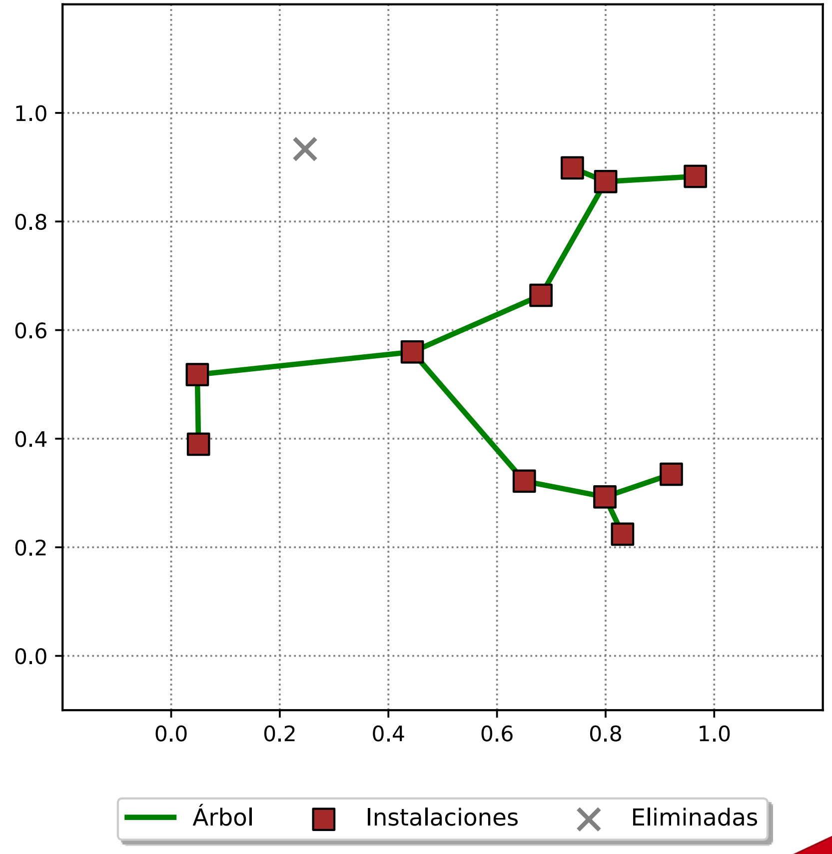
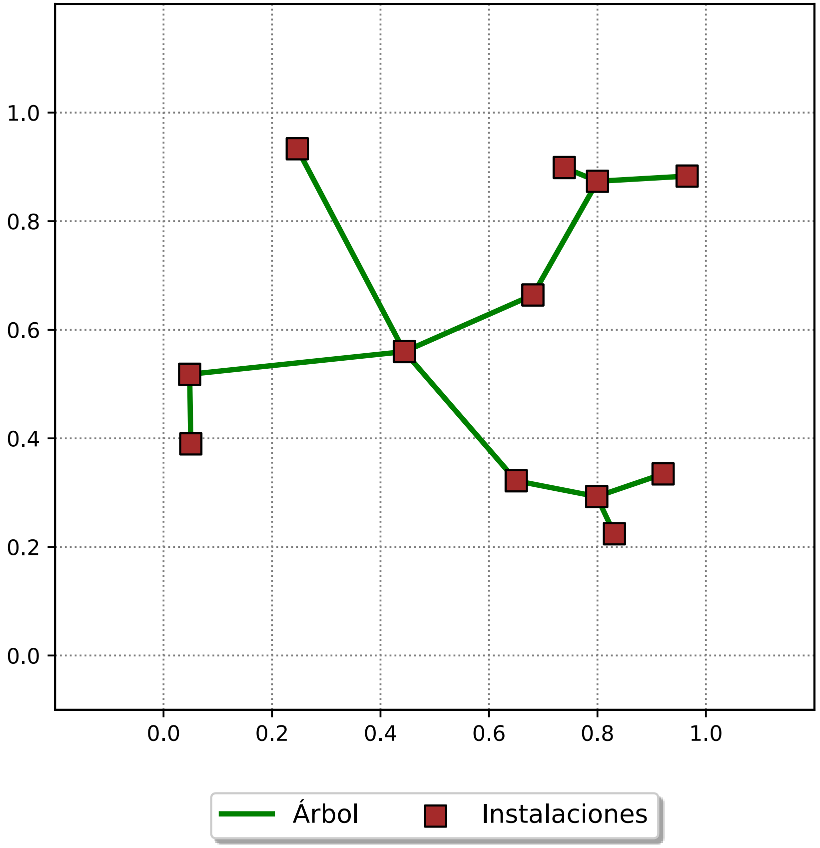


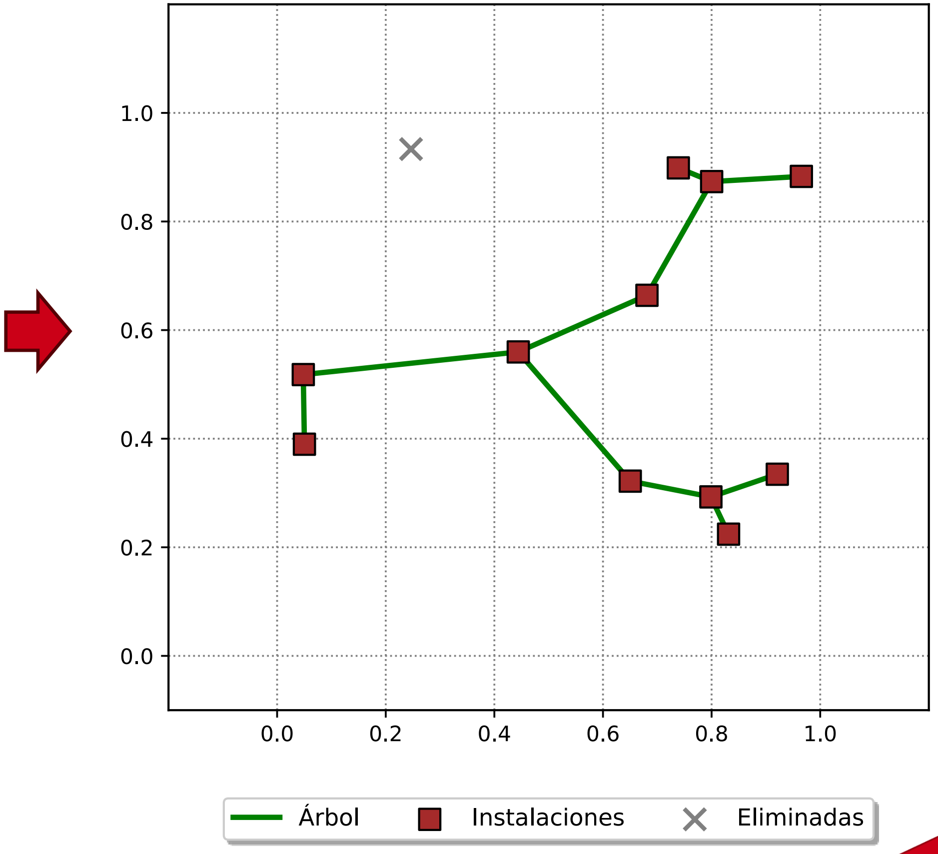


Dynamic MST updates

1) Leaf node

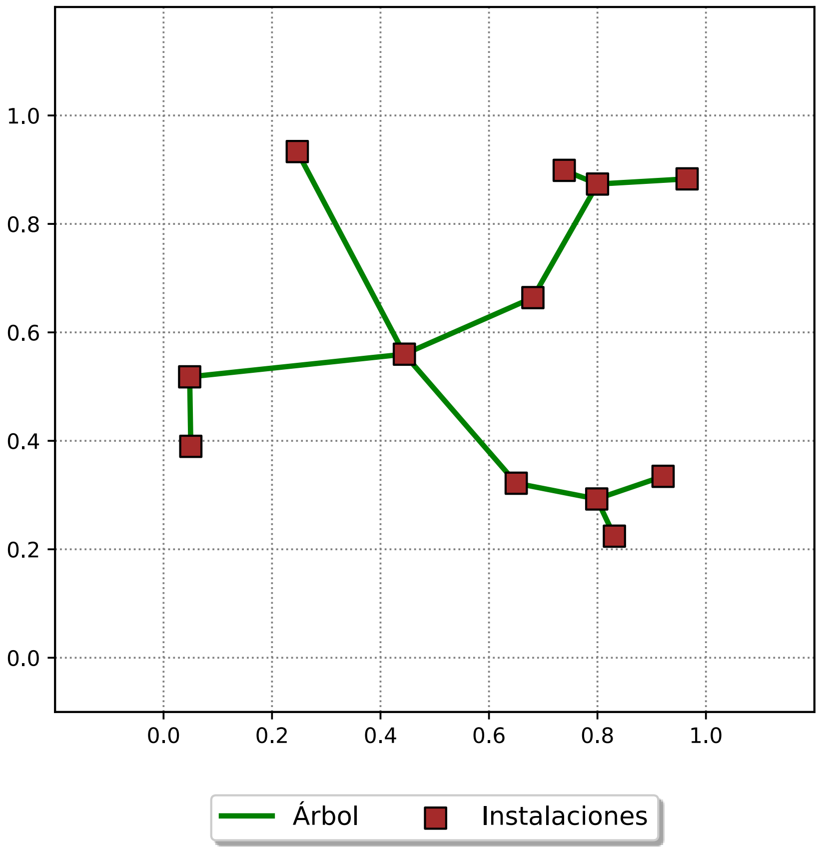
- $O(p^2) \rightarrow O(1)$





Dynamic MST updates

2) Unique connected component



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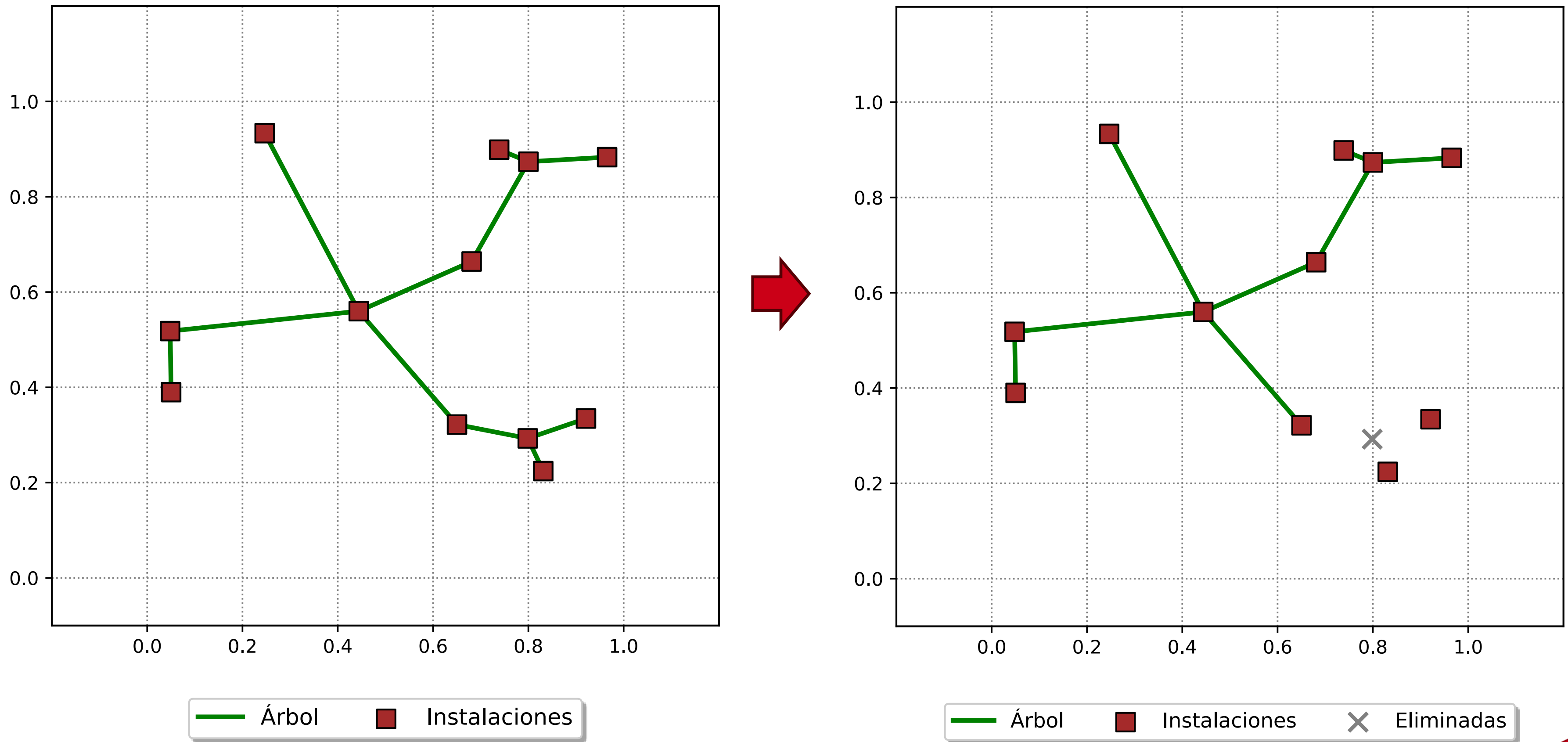

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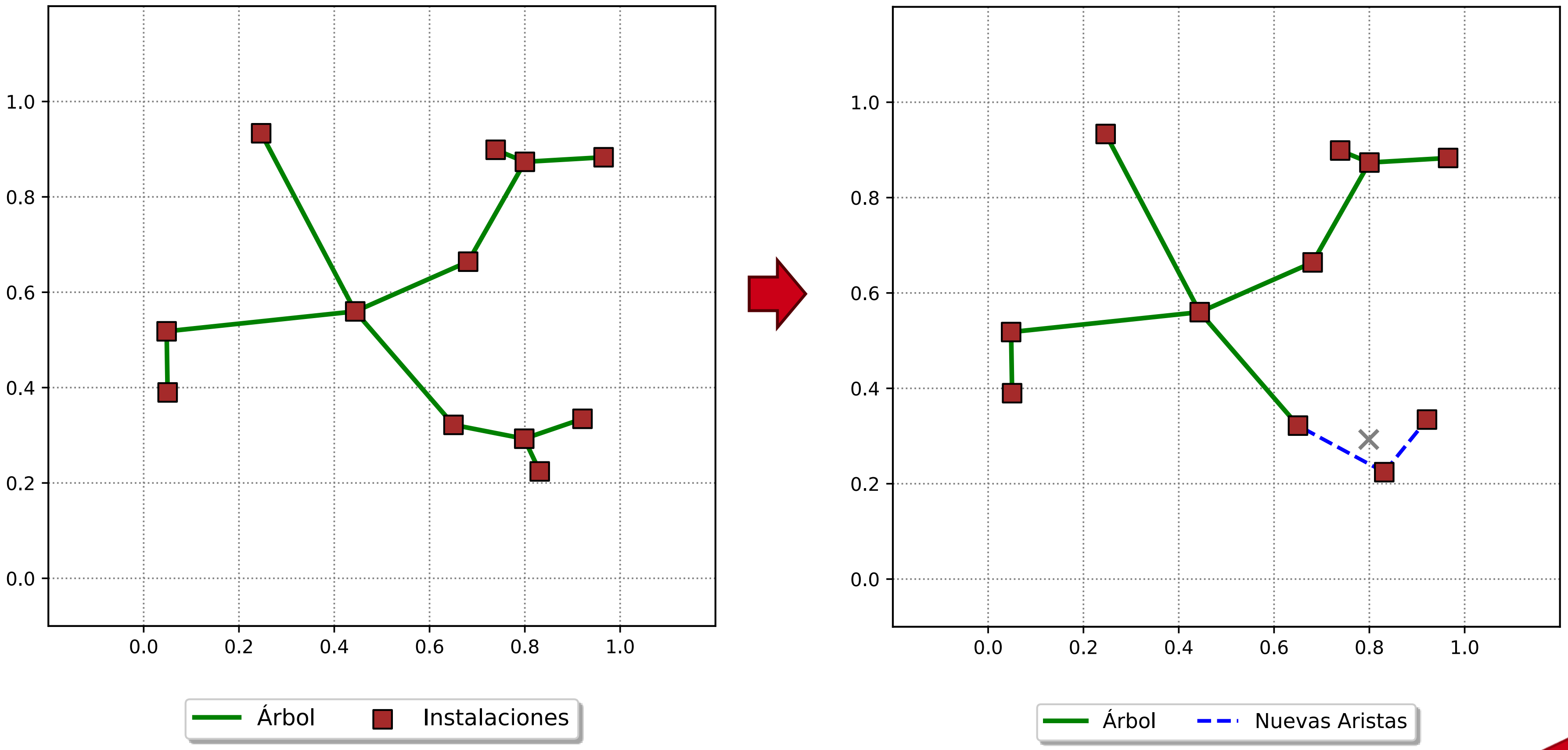

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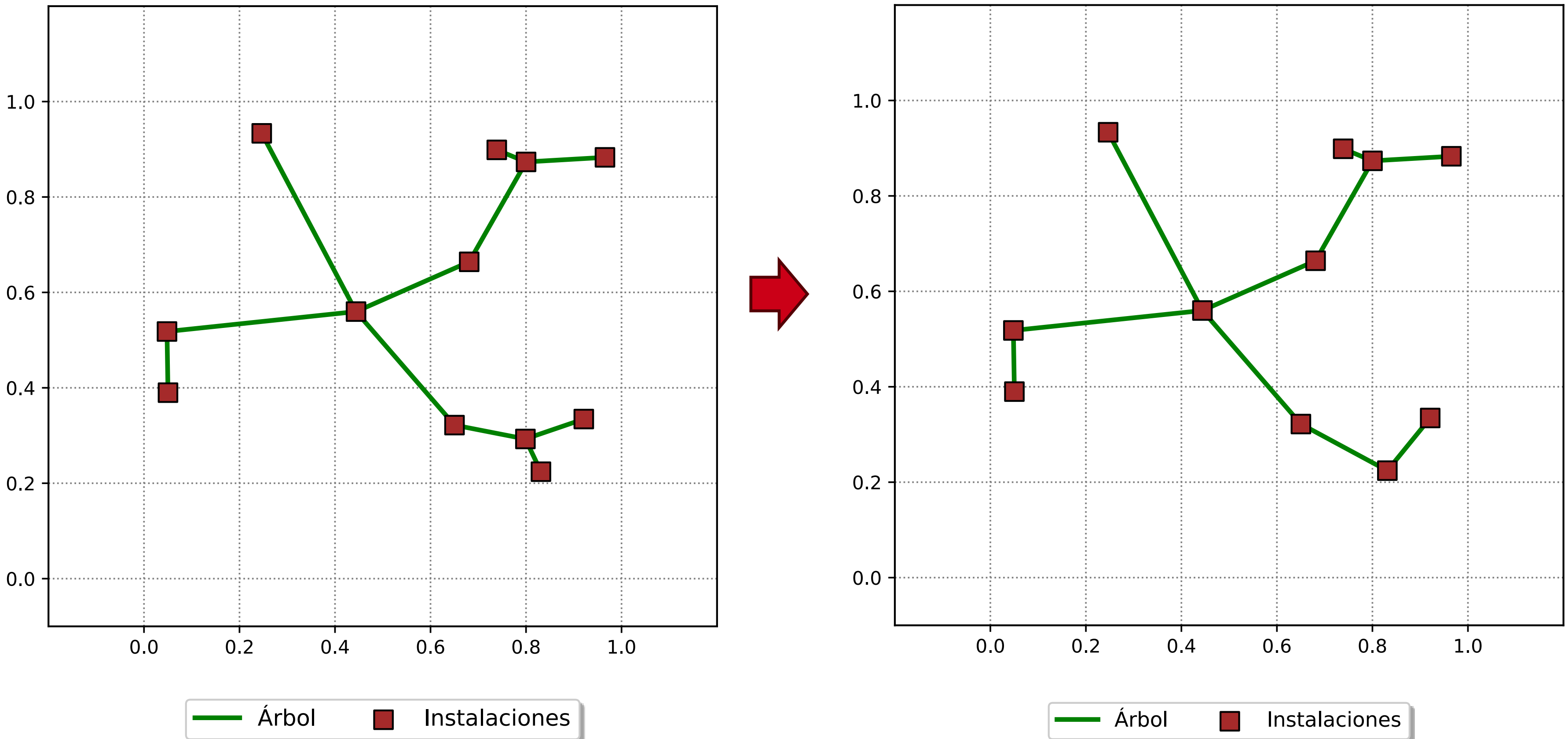
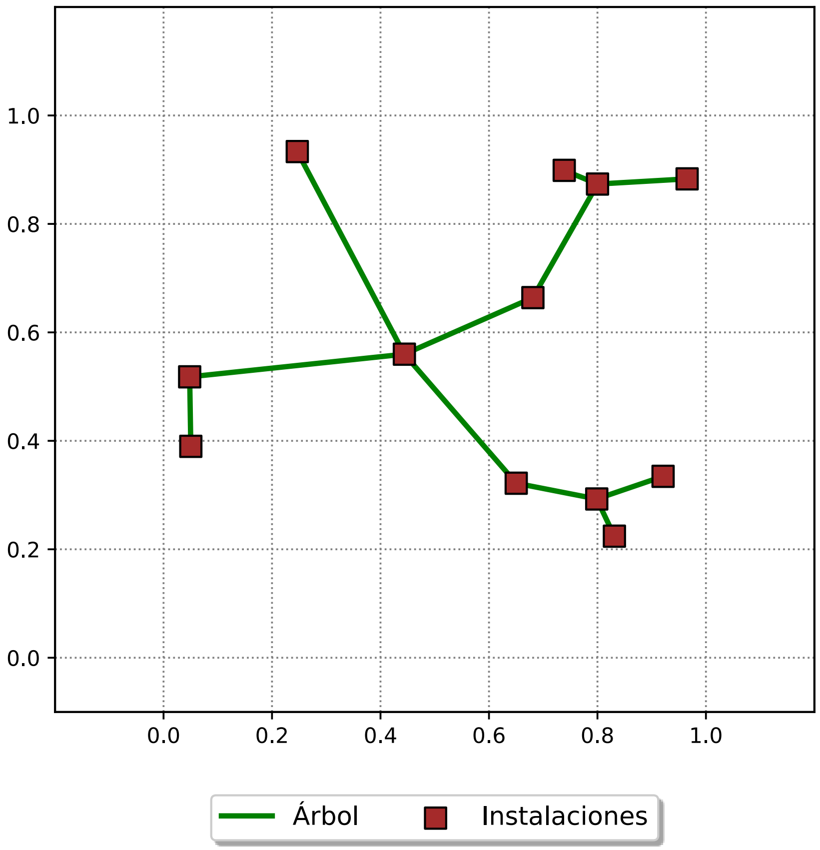
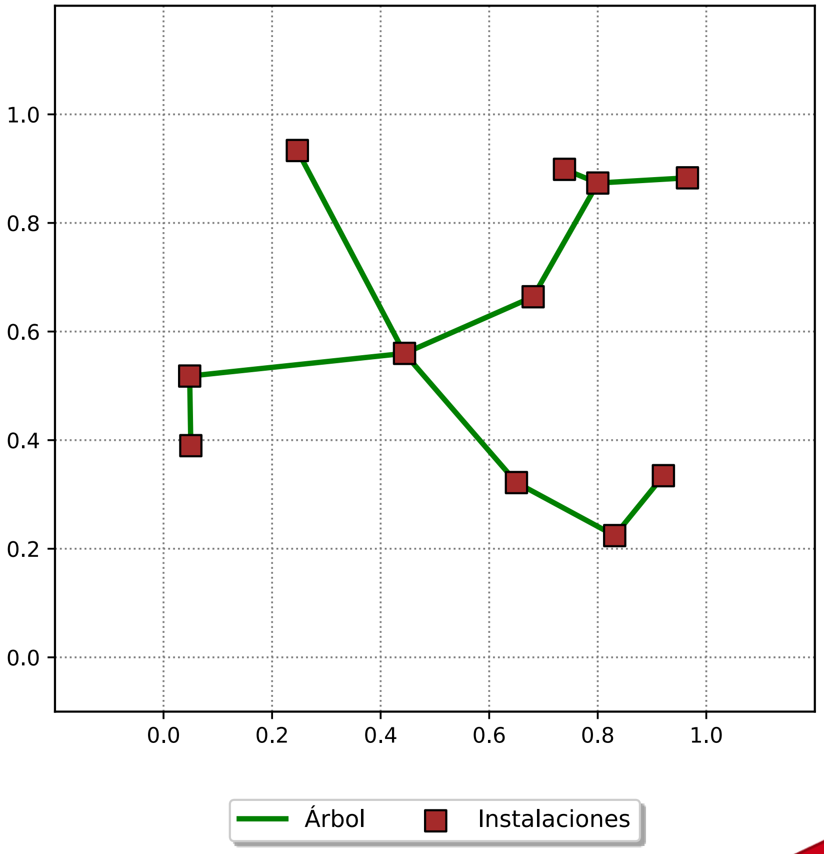


Dynamic MST updates

2) Unique connected component

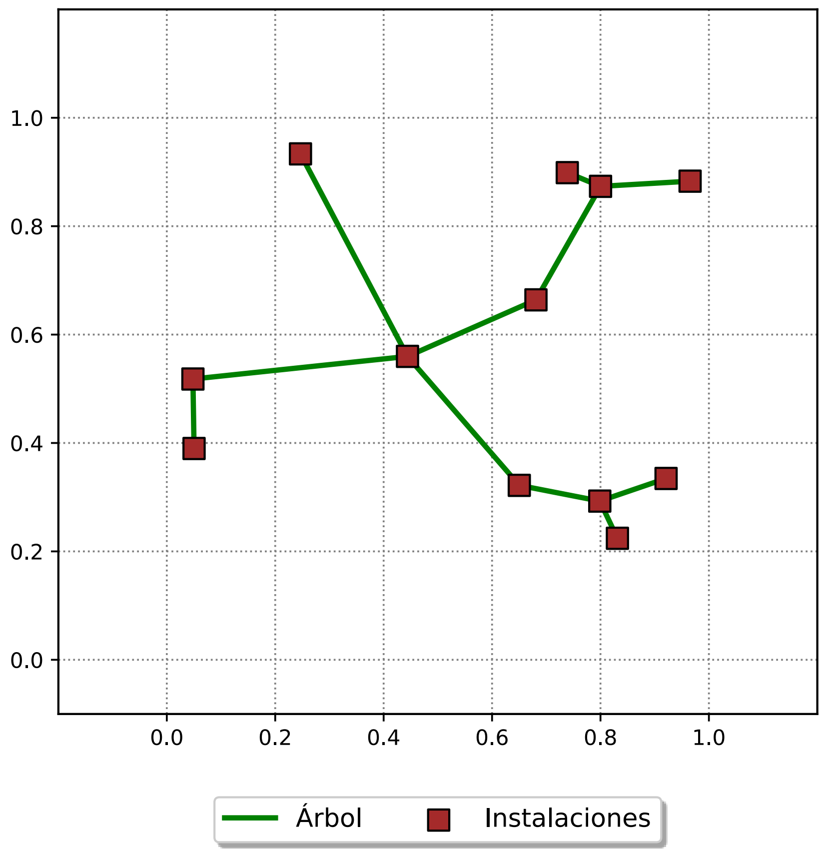
- $O(p^2) \rightarrow O(K \cdot p)$





Dynamic MST updates

3) General Case



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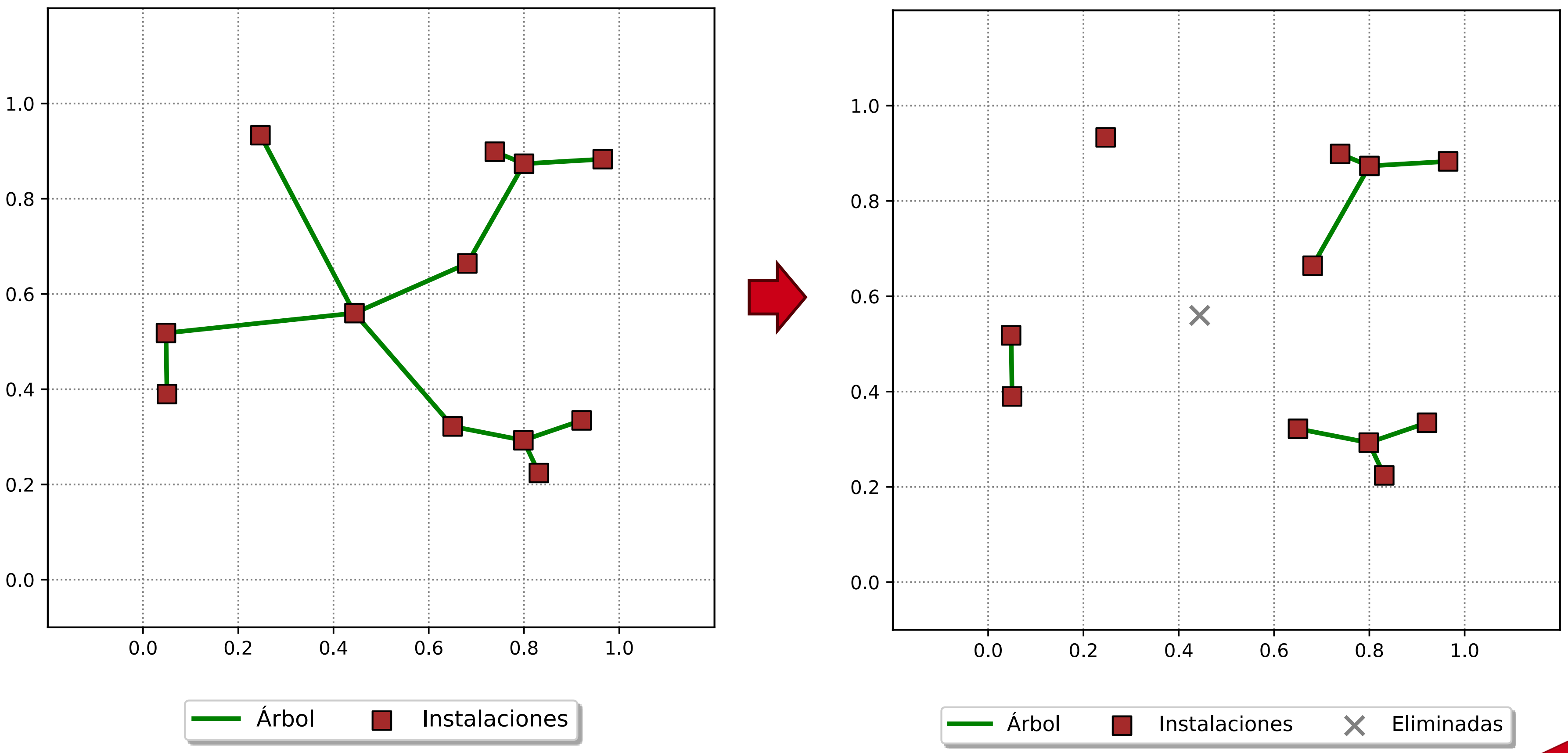

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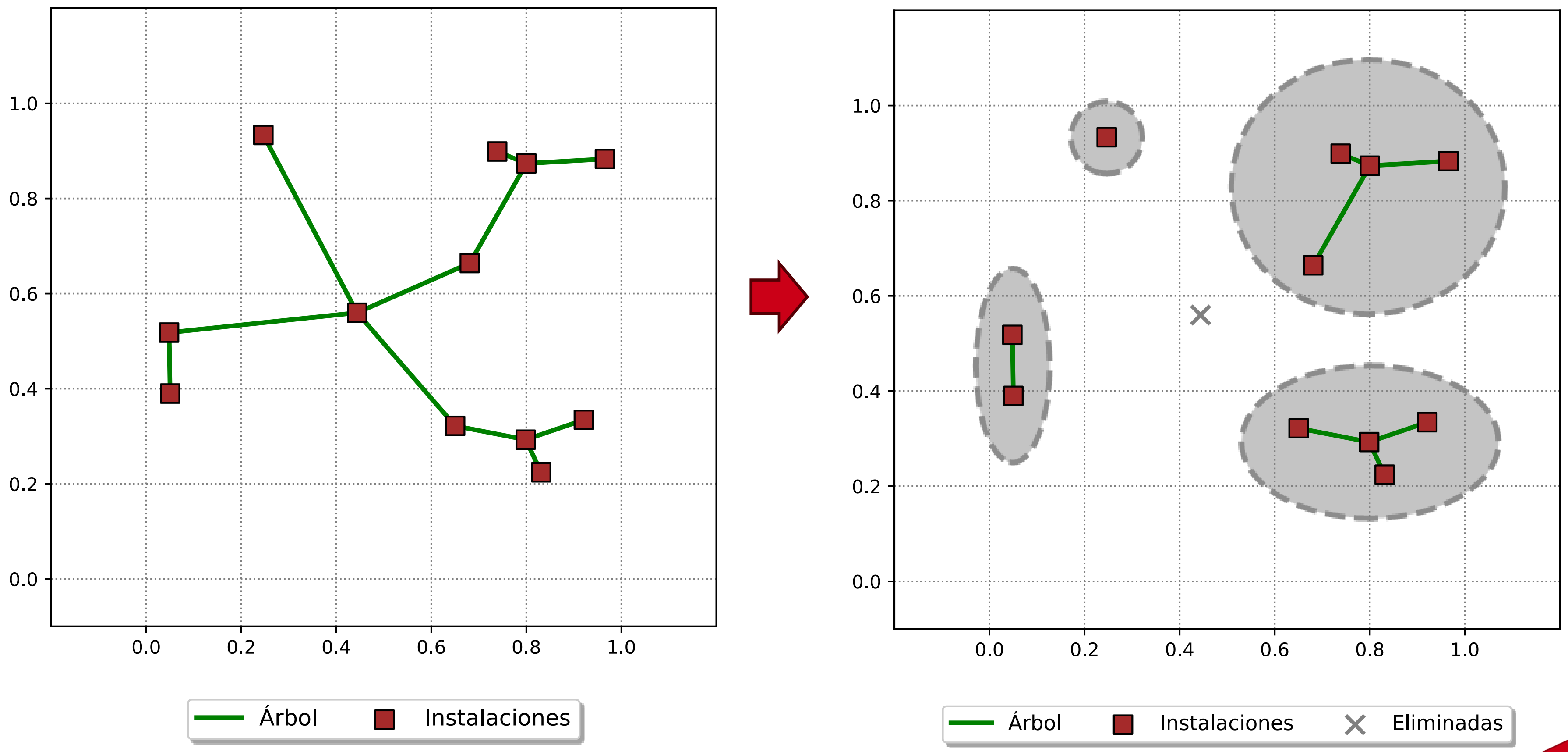

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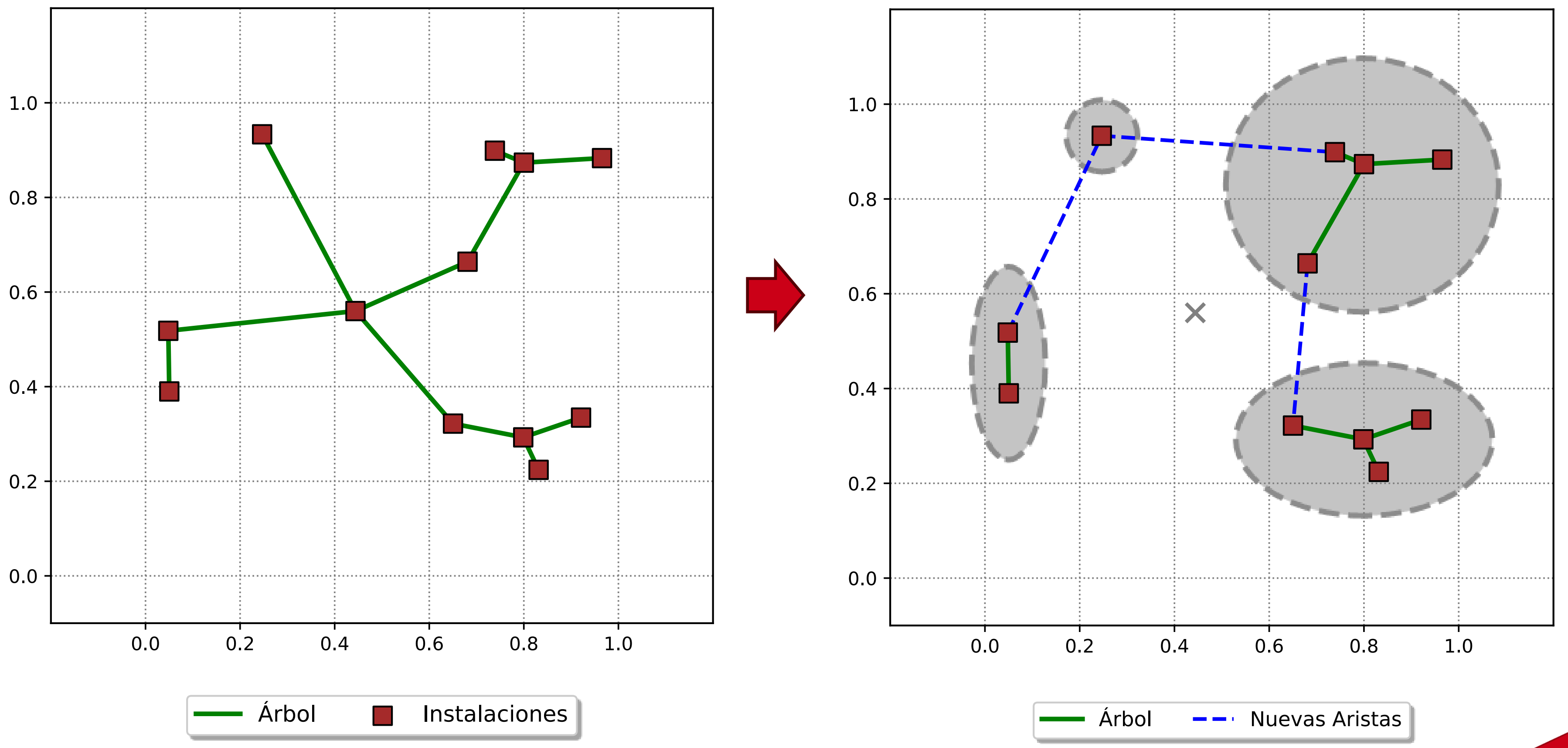

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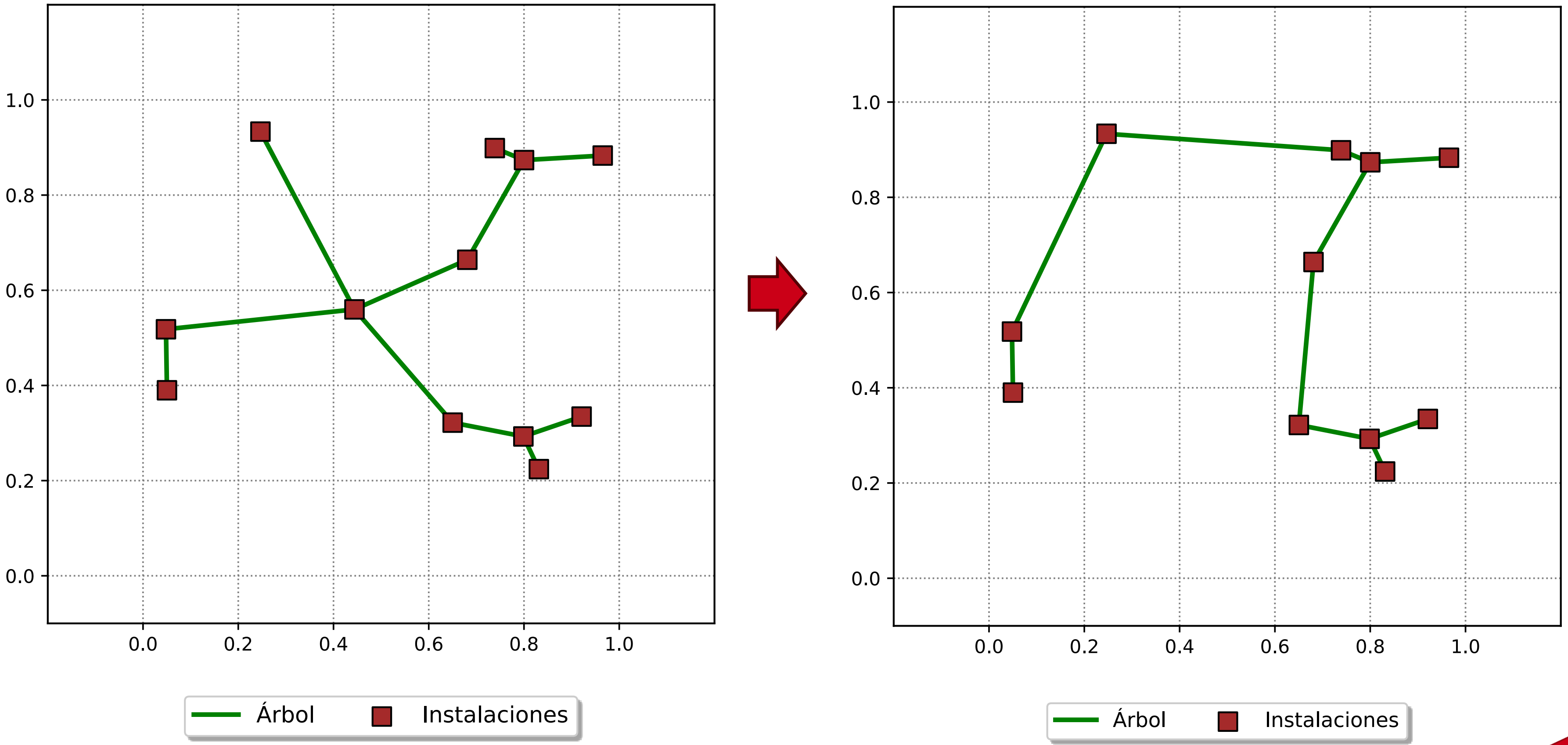

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